Journal of the Civil Engineering Forum, May 2023, 9(2):181-194
DOI 10.22146/jcef.6631
Available Online at https://journal.ugm.ac.id/v3/JCEF/issue/archive

Methods to Determine Ductility of Structural Members: A Review

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SUBMITTED 5 January 2023 **REVISED** 11 February 2023 **ACCEPTED** 7 March 2023

ABSTRACT Ductility plays a crucial role in ensuring the safety of a structure, as its inadequacy can lead to sudden and brittle failure. Despite its significance, there is no explicit method for determining, leading to inconsistency and confusion in selecting appropriate techniques. Misjudging a structure's ductile behaviour can have catastrophic consequences. Therefore, this study examined several preliminary studies and identified twenty-one methods for computing ductility indices. These indices were categorized into three types, namely conventional, displacement-based, and energy-based. The conventional ductility indices are commonly applied to steel-reinforced members, deformation-based ductility indices to FRP-reinforced members, and energy-based ductility indices to earthquake-resistant and static-load structures. Conventional ductility indices are specific to ductile reinforcements, while displacement-based and energy-based ductility indices apply to both ductile and non-ductile reinforcements. However, different calculation methods can lead to significant variations in the computed ductility, particularly for those involving the first crack, and load factor, thereby leading to different ductility requirements for ensuring structural safety. Additionally, not all methods are explicit, and it is crucial to avoid indiscriminately applying requirements from one method to another.

KEYWORDS Ductility Ratio, Structural Element, Reinforced Concrete, Elastic, Plastic

1 INTRODUCTION

The importance of ductility cannot be overstated when it comes to structural safety due to its ability to warn of impending failure (Wang and Belarbi, 2011). According to El Zareef and El Madawy (2018), a structure that exhibits ductility can deform significantly prior to failure. Additionally, ductility allows a structure to absorb and dissipate substantial amounts of energy before failure (Park, 1988), which is crucial for structures in areas with a high risk of earthquakes (Muralidhara Rao et al., 2015). The quantification of ductility is currently not standardized due to the unavailability of explicit method to determine the process (Nogueira and Rodrigues, 2017). Preliminary studies employed a range of methods to assess ductility, with similar parameters but varying computational approaches. The lack of consistency between methods can result in a structure being evaluated with differing degrees of ductility, depending on the method utilized.

In practice, most studies employed one method to determine ductility because it was deemed suitable. Research by Barrera et al. (2012) and Spadea et al. (2001) used two or more methods. Several types of ductility methods have been demonstrated in previous studies (Tann et al., 2004; Ghallab, 2014; Zou, 2003; El Zareef and El Madawy, 2018; Oudah and El-Hacha, 2012). However, only a few common methods have been identified, and many previously used ductility methods remain unidentified. This study aims to provide a comprehensive summary of the different ductility methods available in the literature, along with their computations and applications. Additionally, it aims to highlight the ductility requirements for structures. The primary goals of this study are (a) to provide readers with an understanding of the range of ductility methods available and (b) to assist in selecting suitable methods for elements and structures under varying conditions.

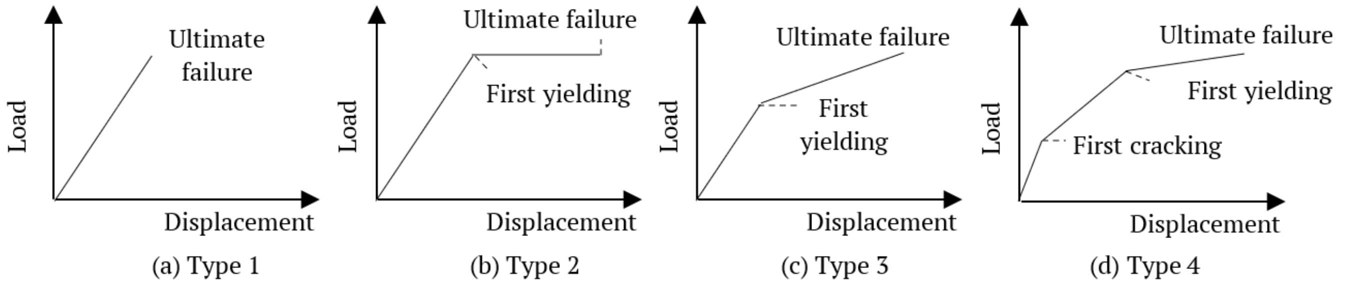

Figure 1 Typical load-displacement curves of structural elements

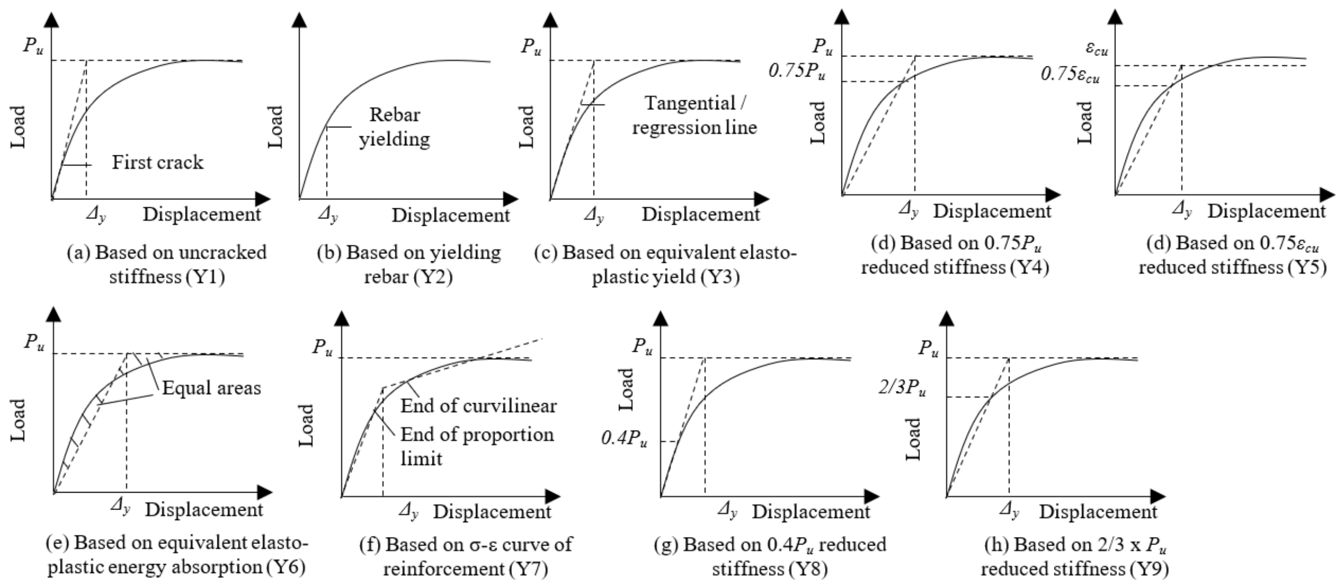

Figure 2 Alternative definitions for yield displacement

2 BEHAVIOUR OF CONCRETE MEMBER

Concrete members can be reinforced by various methods, such as using steel bars, prestressing tendons, and fibre-reinforced polymer (FRP). Each of these reinforcements has unique characteristics that significantly impact the behaviour of the members.

Under load, concrete members undergo displacement, rotation, or curvature. Figure 1 depicts the typical load-displacement responses of a member. Elastic behavior is shown in Figure 1(a), where displacement is proportional to the load throughout. This brittle response is prevalent in concrete members reinforced with fibre-reinforced polymer (FRP) (Abdelraham et al., 1995). Figures 1(b) and 1(c) show the yielding responses of a member with and without post-yield stiffness. When a member yields, its stiffness decreases dramatically, and a large deformation occurs. This post-yield deformation contributes significantly to the member's ductility, and can be observed in RC and PC members (Lim et al., 2021; Ghallab, 2014). Figure 1(d) demonstrates the cracking response of a member before yielding. Cracks appear when the concrete's tensile strain limit is exceeded, reducing the bearing area of the cracked section and subsequently deteriorating its section inertia (Fu et al., 2020). This slightly reduces the member's stiffness (Ling et al., 2019), which further decreases as reinforcements yield (Wang et al., 2021).

3 METHODS TO DETERMINE DUCTILITY

3.1 Conventional Ductility Ratios

The ductility ratio, which is used to measure a member's ductility, can be expressed as a function of curvature, rotation, displacement, or twist, as shown in Table 1. Historically, it was defined based on the behavior of the reinforcement, as described in studies by Zou (2003); Dancygier and Berkover (2016); Ashour (2000). However, this approach is applicable conditional on (a) the reinforcement yields before the member fails and (b) its yield point accurately determined.

The concept of yield deformation has been expanded for larger applications. Figure 2 shows that the yield deformation can be determined based on (a) the cracking response of the concrete (method Y1), (b) the stress-strain response of the reinforcement bar (methods Y2 and Y7), (c) the equivalent stiffness of the member (methods Y3, Y4, Y8, and Y9), (d) the compressive strain limit of the concrete (method Y5), and (e) the elastoplastic energy absorption principle (method Y6).

The yield point can then be determined by (a) constructing a tangential or linear regression line over the elastic part of the curve (method Y3), (b) drawing straight lines intercepting the critical points on the curve (methods Y1, Y4, Y8, and Y9), (c) identifying a point on the curve where the elastic stiffness has decreased by more than 5% (ASTM E2126, 2011), and (d) constructing a straight line that gives equal areas above and below the curve (method Y6). It is important to note that these yield points are hypothetical and that the reinforcement may not necessarily yield because they sometimes remain in the elastic state of the member. This may not be a member's true response, but it can be a conservative estimate.

Figure 3 illustrates several methods for obtaining the ultimate displacement, Δu. These include (a) considering the compressive strength limit of the concrete (U_1), (b) determining the peak load of the member (U_2), (c) analyzing a certain percentage of reduced load after the peak (U_3 and U_4), and (d) identifying the failure of the reinforcement, such as the fracture of the transverse or longitudinal reinforcement or the buckling of the longitudinal compression reinforcement (U_5).

The ductility ratio is obtained by dividing the ultimate displacement, Δu, by the yield displacement, Δy. Table 2 outlines at least 12 combinations of Δu and Δy that researchers have used to determine the ductility of RC and PC members. Different methods may produce varying values for the same member. For that, the ductility ratio is regarded as an indication of ductile behavior. The absolute value is less significant than the relative comparison among members (Tann et al., 2004). The comparison should be made using the same method to ensure consistency.

A precedent was established in preliminary studies using the ductility methods listed in Table 2. However, this list may not be comprehensive, and other methods could be appropriate when properly justified. Tables 3 and 4 outline the circumstances in which the methods are applicable.

3.2 Deformation-Based Ductility Indices

The conventional ductility ratios outlined in Table 2 are applicable for members with clear plastic deformation, specifically Types 2, 3, and 4 in Figure 1. This plastic deformation typically results from the yielding of steel reinforcement (Wang and Belarbi, 2011). However, these ratios are not suitable for concrete members with FRP reinforcement that remain elastic throughout (Type 1 in Figure 1) (Zou, 2003; Grace et al., 1998; Wang and Belarbi, 2011; Abdelraham et al., 1995; El Zareef and El Madawy, 2018). In response, deformation-based ductility indices were introduced and are outlined in Table 5. These indices use more complex equations than the conventional ductility ratios and are expressed in deflection, rotation, and curvature.

The ductility indices compare a member's ultimate state against its serviceability. The ultimate deformation corresponds to the peak load, as shown in Figure 4. The serviceability deformation can be any of the following illustrated in Table 5.

1. for a beam that fails by concrete crushing, the concrete strain at the top compression fibre is about 0.001 (D13),
2. the propagation of the first crack (D14 and D15),
3. 2/3 of the peak load (D16).

Ductility indices D_{13}, D_{14}, and D_{17} (Table 5) comprise load factors like $\frac{M_u}{M_{0.001}}$, $\frac{M_u}{M_{cr}}$, and $\frac{M_{max}}{M_y}$. Despite the deformation, the load is considered a part of ductility. However, this differs from the conventional ductility ratios, which only include deformation.

It is worth noting the difference between ductility and deformability, as explained by Oudah and El-

Table 1. Conventional ductility ratio (El Zareef & El Madawy, 2018; Zou, 2003; Teixeira and Bernardo, 2018)

Ductility ratio	Application	Equation	Description
Curvature ductility, μ_ϕ	element loaded by a bending moment	$\mu_\phi = \frac{\phi_u}{\phi_y}$ (1)	a) ϕ_u is the curvature of the element at the ultimate state b) ϕ_y is the curvature of the element at the yield limit
Rotation ductility, μ_θ	plastic hinge loaded by a bending moment	$\mu_\theta = \frac{\theta_u}{\theta_y}$ (2)	a) θ_u is the ultimate plastic hinge rotation b) θ_y is yield plastic hinge rotation
Displacement ductility, μ_Δ	structural element or structure loaded by one or more forces	$\mu_\Delta = \frac{\Delta_u}{\Delta_y}$ (3)	a) Δ_u is the displacement of a structural element or a whole structure at the ultimate state b) Δ_y is the displacement at the yield limit
Torsional ductility, μ_τ	Structural element loaded by torsional load	$\mu_\tau = \frac{\theta_{\tau,u}}{\theta_{\tau,y}}$ (4)	a) $\theta_{\tau,u}$ is the ultimate twist b) $\theta_{\tau,y}$ is the yielding twist

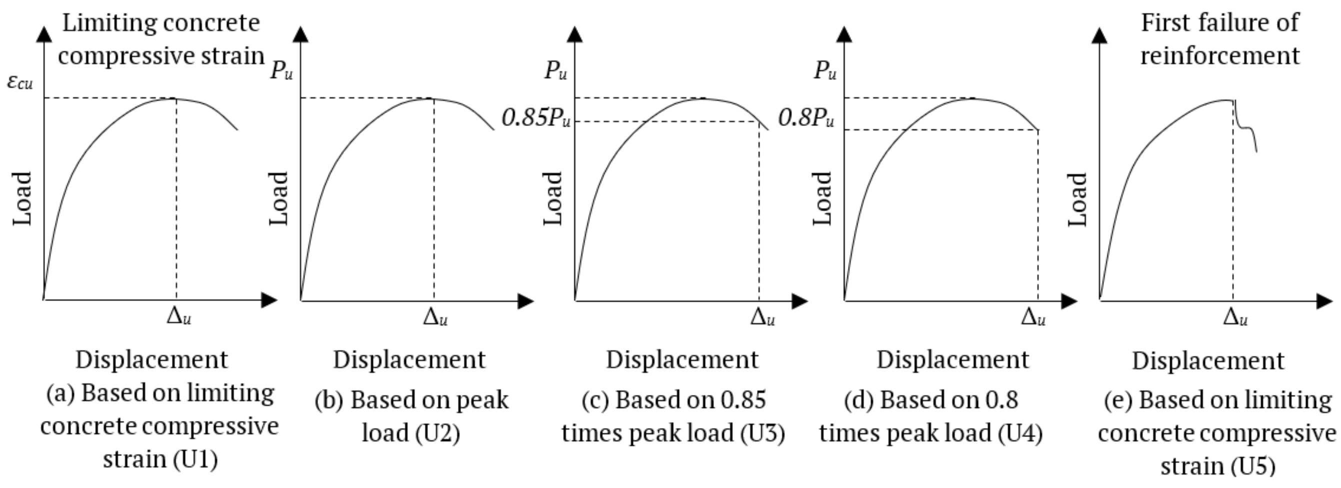

Figure 3 Alternative definitions for ultimate displacement

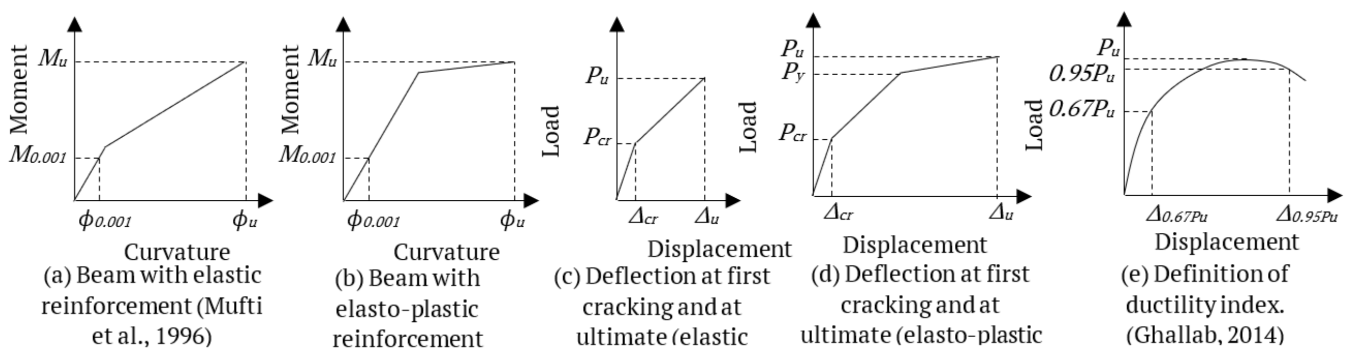

Figure 4 Response of structural member under load

Table 2. Methods to compute conventional ductility ratios

	Yield displacement									Ultimate displacement					Reference
	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	U1	U2	U3	U4	U5	
D1	√										√				Abdelraham et al. (1995); Dolan et al. (2001)
D2		√									√				Ashour (2000); Dolan et al. (2001); Tann et al. (2004)
D3			√								√				Naaman (1985); Soudki (1994); NZS 3101.1 (2006)
D4			√									√			Rakhshanimehr et al. (2014); Chen and Sudibyo (2018)
D5				√								√			Jang et al. (2008); **?**); Pam et al. (2001)
D6				√									√		Kwan et al. (2002); Wu (2006)
D7					√						√				Naaman (1985)
D8						√						√			Nie et al. (2014)
D9							√				√				Naaman (1985)
D10								√					√		International (2011)
D11									√		√				Pan and Moehle (1989)
D12		√	√	√		√				√	√	√	√	√	Park (1988)

Table 3. Determining the yield displacement of a member under various circumstances

Circumstances	Methods to determine yield displacement								
	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
The first cracking load is measured	√								
The reinforcement yielded before the member fails.		√					√		
The stress-strain curve of the reinforcement is available.		√					√		
The member fails by crushing					√				
The member is predominantly subjected to compressive force					√				
The stress-strain curve of concrete is available					√				
The member subjected to cyclic load						√			
Others			√	√		√		√	√

Table 4. Determining the ultimate displacement of a member under various circumstances

Circumstances	Methods to determine ultimate displacement				
	U1	U2	U3	U4	U5
The stress-strain curve of concrete is available	√				
The load-displacement response after peak load is available			√	√	
The failure of reinforcement is detected					√
Others		√			

Table 5. Deformation-based ductility indices

Method	Equation	Description	
D13	$\mu_M = \frac{M_u}{M_{0.001}} \cdot \frac{\phi_u}{\phi_{0.001}}$ or $\mu_M = \frac{M_u}{M_{0.001}} \cdot \frac{\Delta_u}{\Delta_{0.001}}$	1. μ is the ultimate moment 2. ϕ_u is the curvature at the ultimate state. 3. $\phi_{0.001}$ is the curvature when the concrete strain at the outermost compression fiber is 0.001 (Figures 4(a) and 4(b)). 4. $M_{0.001}$ is the moment the concrete strain at the outermost compression fiber is 0.001.	Mufti et al. (1996); Bakht et al. (2000); Canadian Standards Association (2006); Jaeger et al. (1997); Thériault and Benmokrane (1998)
D14	$\mu_z = \frac{\Delta_u}{\Delta_{cr}} \cdot \frac{M_u}{M_{cr}}$	1. Δ_u is deflection at ultimate. 2. Δ_{cr} is deflection at first cracking (Figure 5). 3. M_u is the ultimate moment. 4. M_{cr} is the cracking moment.	Zou (2003)
D15	$\mu_d = \frac{\Delta_u - \Delta_{cr}}{\Delta_{cr}}$	1. Δ_u is deflection at ultimate. 2. Δ_{cr} is deflection at cracking load (Figure 5).	Hassan et al. (2018)
D16	$\mu_t = \frac{\Delta_{0.95Pu}}{\Delta_{0.67Pu}}$	1. $\Delta_{0.95Pu}$ is the displacement at 95% peak load (Figure 6). 2. $\Delta_{0.67Pu}$ is the displacement at 67% peak load.	Ghallab (2014); Tann et al. (2004)
D17	$\mu_\phi = \frac{\phi_u}{\phi_y}$	1. ϕ_u is curvature when (a) the post-peak remaining moment capacity of the column reduces to 80% of the maximum moment capacity M_{max}, or (b) the longitudinal reinforcement steel strain reaches the ultimate strain ϵ_{su}, or (c) the strain in concrete reaches the maximum confined strain ϵ_{ccu}. 2. $\phi_y = \min\left(\phi_y \frac{M_{\max}}{M_{yc}}, \phi_{ys} \frac{M_{\max}}{M_{ys}}\right).$ 3. ϕ_{yc} is the curvature when the concrete strain reaches the strain at peak stress in unconfined concrete 4. M_{yc} is the moment corresponding to ϕ_{yc}. 5. ϕ_{ys} is the curvature at the onset of the first yielding of the longitudinal bars. 6. M_{ys} is the moment corresponding to ϕ_{ys}.	Paultre and Légeron (2008)

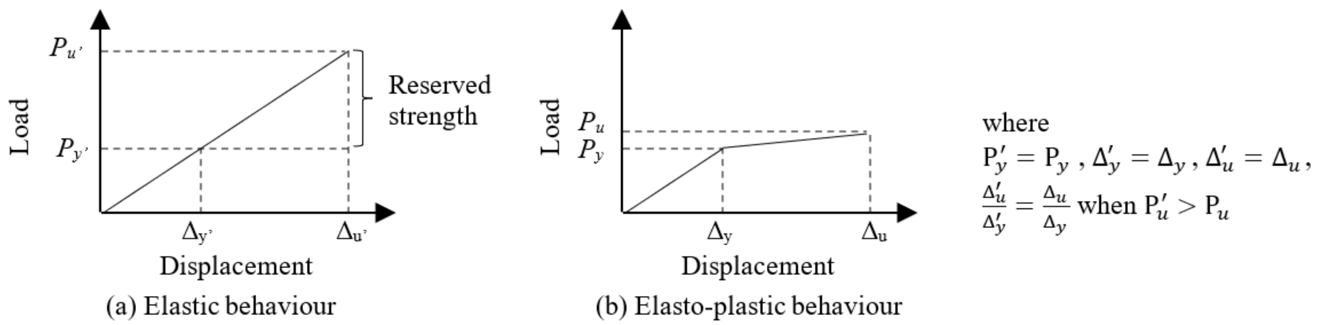

Figure 5 Ductility of the members with elastic and elastoplastic reinforcements

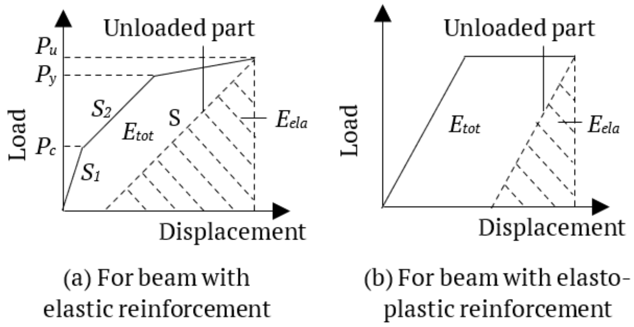

Figure 6 Total and elastic energy under the load-deflection curve (Naaman and Jeong, 1995)

Hacha (2012); Tann et al. (2004). Ductility refers to the plastic work that a member can undergo before it fails, and it requires yielding and plastic behavior. In contrast, deformability refers to the amount of deformation that a member can experience before failure, regardless of whether it exhibits yielding and plastic behavior. Several methods, including D1, D13, D14, D15, and D17, are considered deformability indices. They compute either the member's first crack load or the concrete's elastic strain (i.e., 0.001) without necessarily requiring plastic failure.

According to Tann et al. (2004), high deformability is a necessary but insufficient prerequisite for ductile behaviour. Although a ductile member typically exhibits high deformability, a member with high deformability can still experience brittle failure (Tann et al., 2004). Nevertheless, a highly deformable member can provide early warnings before failure by showing significant elastic deformations. For adequate ductility, an elastic member should possess a higher reserve of strength than a ductile member (ACI Committee 440, 2015), as shown in Figure 5.

3.3 Energy-Based Ductility Indices

The energy-based ductility indices adopt the concept of energy in ductility. They apply to structures subjected to earthquake loads. These ductility indices can also be used for members subjected to static loads (Antonius and Imran, 2012; Hason et al., 2021). The ductility indices typically deal with a member's total energy at ultimate and its energy at service, as shown in (Table 6). The area beneath the load-displacement curve represents the total energy possessed by a member (Figures 6, 7, and 8). The energy at service can be one of the following: (a) the area under the elastic region of the load-displacement curve (method D18), (b) the area under the load-displacement curve up to 0.75 times the ultimate load (method D19), (c) the area under the load-displacement curve at the yielding of tension reinforcement (method D20), and (d) the area under the load-displacement curve at service (method D21).

The method D17 was originally introduced by Naaman and Jeong (1995) as a means of analyzing RC members subjected to cyclic loading. In this method, the inelastic energy and elastic energy are separated by a line, as shown in Figure 6. Its slope, S, is computed using equation E1 in Table 7, which covers only the member's load response. Grace et al. (1998) further included the effects of reinforcement, failure mode, and stirrup in the function as given in equation E2 (Table 7).

4 DUCTILITY REQUIREMENTS

Ductility plays a crucial role in the integrity of a structure. Compared to non-ductile structures, ductile structures possess the ability to withstand unexpected or unforeseen forces more effectively

Table 6. Energy-based ductility indices

Method	Equation	Description	Reference
D18	$\mu_{en} = 0.5\left(\frac{E_{tot}}{E_{ela}} + 1\right)$ (5)	a) E_{tot} is the total energy, which is the area under the load-displacement curve up to the failure load (Figure 6) b) E_{ela} is the elastic energy computed as the area of the triangle formed at failure load by unloading the beam (Figure 6)	Naaman and Jeong (1995); Grace et al. (1998); Tann et al. (2004)
D19	$\mu_e = \frac{E_{tot}}{E_{0.75pu}}$ (6)	a) E_{tot} is the total area under the load-displacement curve at ultimate failure (Figure 7) b) $E_{0.75pu}$ is the area under the load-displacement curve up to 0.75 times the ultimate load (Figure 7)	Spadea et al. (1997); Alsayed and Alhozaimy (1999)
D20	$\mu_e = \frac{E_{tot}}{E_y}$ (7)	a) E_{tot} is the total area under the load-displacement curve at ultimate failure (Figure 8) b) E_y is the area under the load-displacement curve at the yielding of tension steel (Figure 8)	Spadea et al. (2001); Thomsen et al. (2004)
D21	$\mu_e = \frac{E_{tot}}{E_{y,s}}$ (8)	a) E_{tot} is the area under the load-displacement curve at ultimate (Figure 8) b) $E_{y,s}$ is the area under the load-displacement curve at service (Figure 8)	Ghallab (2014); ACI Committee 440 (2015)

Table 7. Equations for the slope of the unloading branch to compute method D17

	E1	E2
Reference	Naaman and Jeong (1995); Grace et al. (1998)	
Function	$S = \frac{P_c S_1 + (P_y - P_c) S_2}{P_y}$ (9)	$S = \frac{\alpha\rho\gamma(E_f f_y)}{(E_s f_{ds})} \cdot \frac{P_c S_1 + (P_y - P_c) S_2 + (P_u - P_y) S_3}{P_y}$ (10)
Description	• S is the slope of the unloading branch • S1 is the slope of the first line • S2 is the slope of the second line • P_c is the cracking load • P_y is the yielding load.	• α is the factor related to the stirrup type effect, • ρ is the factor related to the failure mode effect, • γ is the factor related to the type of reinforcement effect • Ef is the FRP modulus of elasticity, • Es the steel modulus of elasticity • fy the steel yield stress • fds the design strength of FRP • Pu the ultimate load, • S1 is the slope of the first line • S2 is the slope of the second line • S3 is the slope of the third line

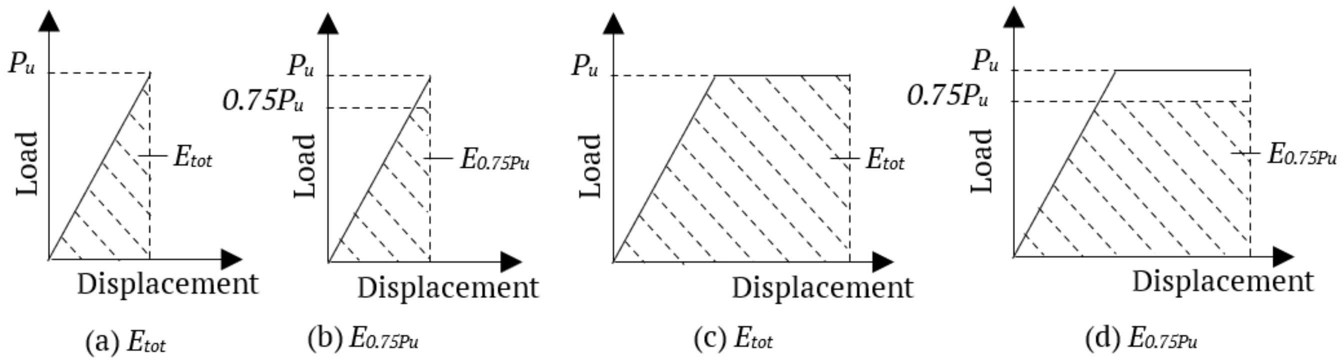

Figure 7 Energy under the load-deflection curve (Spadea et,al., 1997)

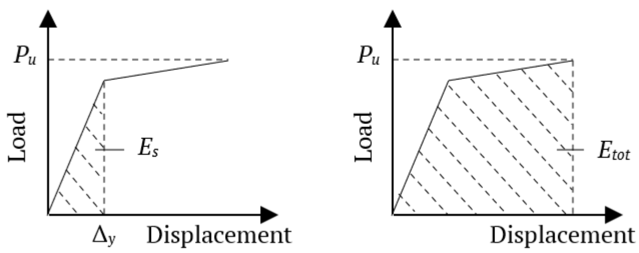

Figure 8 Energy under a load-deflection curve

(Ghallab, 2014). A ductile structure can (a) deform significantly prior to failure (El Zareef and El Madawy, 2018), (b) redistribute moments during excessive loads (Ashour, 2000), and (c) dissipate substantial amounts of energy prior to collapse (Park, 1988).

A member should satisfy ductility requirements for it to be used reliably in structural engineering applications (Naaman, 2003). The degree of ductility required can vary depending on the type of structure. NZS 3101.1 (2006) outlines the structural ductility factors, as per method D3, for different types of structures, as shown in Table 8. Structures with high ductility can better withstand seismic activities, whereas brittle structures require primary seismic-resisting members to resist earthquake forces.

Applying the structural ductility factors specified in NZS 3101.1 (2006) to other ductility indices may not be straightforward, as different calculation methods can yield significantly varying values, as shown in Table 9. Some methods, such as those that consider the first crack (e.g., method D1) and load factors (e.g., methods D13 and D14), can result in overestimated ductility indices, with values exceeding 8 and, in some cases, as high as 11 (Grace et al., 1998). Table 10 presents other ductility requirements found in previous studies utilized to evaluate the ductility of structures.

It is important to note that the ductility requirements for all methods may not be adequately specified in Table 10. As such, the appropriate range for ductile behavior may not be fully determined for these methods. In such cases, it is recommended to compare the ductility indices of a member to those of control specimens.

To achieve sufficient ductility, it is recommended that reinforced concrete (RC) members be under-reinforced to ensure the reinforcement yields before the concrete crushes (Wu, 2006). This can be achieved by having a reinforcement ratio of less than 1.5% (Ashour, 2000). The tensile strength of the reinforcement should be at least 1.1 times or 1.25 times its specified yield strength (ACI-318, 2019; BS-EN 1992-1-1, 2004). Additionally, the ultimate elongation of the reinforcement should be at least 8% (Macchi et al., 1996).

5 CONCLUSION

This study presents a comprehensive overview of 21 different ductility indices that have been identified in the literature. These ductility indices can be broadly categorized into three types, namely conventional, deformation-based, and energy-based. The conventional ductility indices are commonly applied to steel-reinforced members, deformation-based ductility indices to FRP-reinforced members, and energy-based ductility indices to earthquake-resistant and static-load structures. The conventional ductility indices are suitable for members with ductile reinforcements,

Table 8. Classifications of ductile structure (NZS 3101.1: 2006)

Classification	Description	Structural ductility factor
Brittle concrete structures	Structures that contain primary seismic resisting members.	Not specified
Nominally ductile structures	Structures designed with a low structural ductility factor.	⩽1.2
Structures of limited ductility	Structures which are sub-set of ductile structure.	⩽3.0
Ductile structures	Structures designed for high level of ductility.	⩽6.0

Table 9. Value range of ductility indices based on the results given by Zou (2003)

Samples	Nos. of specimen	Methods to determine ductility				
		D3	D17(E1)	D1	D13	D14
Beams tested by Zou (2003) with Aramid Fiber Reinforced Polymer and Steel Tendons	13	2.5-3.7	1.35-4.24	4.5-22.1		10.7-59.5
Beams tested by Zou (2003) with Carbon Fiber Reinforced Polymer and Steel Tendons	8	2.4	1.24-1.81	8.5-12		39.8-67.8
Beams Tested by Jeong (1994)*	5		1.21-1.93	3.34-10.42		23.4-158
Prestressed Beams Tested by Abdelraham et al. (1995)*	10			5.74-25.59	1.65-9.53	54.4-254.6
Beams Tested by Fam (1995)*	6	2.73		4.25-8.03		26-47.1

*Results computed by Zou (2003) from the literature

while the displacement-based and energy-based ductility indices are applicable to both ductile and non-ductile reinforcements.

The computation of ductility indices is typically based on the deformation of a member, including displacement, rotation, curvature, and twist, as well as the energy at different states, such as ultimate, yield, and service. There are 5, 9, and 3 ways to represent the ultimate, yield, and service states, respectively. This lead to significant variations in the calculated values for ductility. It should be noted that certain methods, such as those involving the first crack and load factor, can lead to large ductility index values greater than 8.

This study summarises the ductility requirements for brittle and ductile structures. Only six methods have the ductility requirements explicitly stated and the requirements for one method cannot simply be used for another. This is to avoid misjudging a structure's ductile behaviour, which can be catastrophic.

The focus of this study is on the ductility indices that are computed from load-deformation curves. This excludes those calculated using analytical equations or numerical methods. Future studies may consider examining this aspect in greater detail.

DISCLAIMER

The authors declare no conflict of interest.

ACKNOWLEDGMENTS

This study was supported by the Research Grants of the University of Technology Sarawak, UTS/RESEARCH/2/2022/15/01.

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Table 10. Ductility requirements

Ductility type	Classification / requirement			Reference
Displacement ductility (method D2)	3 to 5 for seismic design			Ashour (2000)
Structural ductility factor (method D3)	a) Ductile structure: 4 to 6 b) Structures of limited ductile: 2 to 3 c) Nominally ductile structure 1.25			NZS 3101.1 (2006)
Displacement ductility (method D3	⩾4 for low to moderate seismicity			Soudki (1994); Soudki et al. (1995)
Displacement ductility (method D11)	a) About 6 for walls b) <2 for slab-column connection			Pan and Moehle (1989)
Ductility index (method D13)	a) ⩾4 for rectangular section b) ⩾6 for T-section			Canadian Standards Association (2006); Jaeger et al. (1997); Wang and Belarbi (2011)
Displacement ductility (method D16)		Experiment*1	Review*2	Tann et al. (2004)
	Ductile	1.93 – 2.09	2.27 - 3.11	
	Near ductile	1.81 – 2.11	1.66 - 2.72	
	Brittle	1.65 – 1.75	1.51 - 1.89	
Energy ductility (method D17 and E1)		Experiment*1	Review*2	Tann et al. (2004)
	Ductile	1.95 – 2.31	2.11-3.27	
	Near ductile	1.62 – 1.77	1.1-2.23	
	Brittle	1.24 – 1.44	1.2-1.53	
Energy ductility (method D17 and E1)	For FRP-strengthened flexural member a) Minimum ductility index = 2 b) desirable value ⩾ 2.5			Tann et al. (2004)
Energy ratio (method D17 and E2)	a) Ductile failure: energy ratio ⩾75% b) Semi-ductile failure: 70% to 74% energy ratio c) Brittle failure: below 69%			Grace et al. (1998)

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