ON LEAF AREA FACTOR DETERMINATION USING LINEAR MEASUREMENTS

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RINGKASAN

Luas permukaan daun sering merupakan parameter yang diinginkan dalam berbagai penelitian fisiologis tanaman. Bermacam metoda telah dikembangkan untuk keperluan itu, yang dapat digolongkan atas metoda langsung, tidak langsung, yang merusak daun dan yang tidak merusak daun. Salah satu metoda tidak langsung dan tidak merusak daun yang sering digunakan karena mudah dan cepat adalah menggunakan rumus matematika yang menghubungkan luas permukaan daun dengan umumnya satu karakter daun yang lain, misalnya panjang atau lebar daun atau kombinasi yang lain. Fungsi yang sering dipakai adalah fungsi liner. Mudah difahami bahwa pemilihan $Y = \beta X + \in$ sebagai model lebih wajar daripada $Y = \alpha + \beta X + \in$, di mana Y = luas permukaan daun, X = karakter daun, β = koefisien regresi atau sering pula disebut faktor luas daun mengingat fungsinya, dan $\in$ = penyimpangan. Alasannya adalah bahwa luas permukaan daun akan sama dengan nol kalau karakter daun yang lain nol pula. Nilai $\Sigma XY / \Sigma X^2$ sebagai penduga dari β merupakan penduga yang tidak bias dengan varian terkecil. Menduga β dengan model $Y = a + bX$ kalau model sebenarnya $Y = \beta X + \in$, akan menghasilkan penduga yang juga tidak bias tetapi variannya lebih besar. Demikian pula menduga β dengan cara menghitung rata-rata faktor luas daun tiap daun yang didapat dari membagi luas daun dengan karakter daun yang lain (Y/X).

SUMMARY

One of the most frequently used nondestructive and indirect method of estimating leaf area is based on the fact that leaf area is a function of one or more dimensions of leaf. Linear function relating leaf area and one dimension of leaf is commonly adopted. It is more reasonable to think of the model $Y = \beta X + \in$ than $Y = \alpha + \beta X + \in$, where Y = leaf area, X = leaf dimension, β = regression coefficient or sometimes called leaf area factor, and $\in$ = error term. The estimation of β and its related statistic is discussed assuming the model $Y = \beta X + \in$ is true but is estimated by $Y = a + bX$ or simply by averaging individual leaf area factor from each leaf from Y/X.

INTRODUCTION

In many crop physiological studies, leaf area measurement is often required. Various method has been developed. Marshall [7] has presented a comprehensive review of the methods and has classified them as destructive, non-destructive, direct, or indirect. Similar descriptions, but in less detail, have been reported by McKee [8]. One of the most frequently used nondestructive and indirect method, due to less time consuming and convenience, is that of estimating leaf area from mathematical formula relating leaf area and one or more dimensions of leaf. Such method has been applied to numerous crop plants : corn [5, 8], soybean [13], cotton [1], sunflower [10], safflower [11], sorghum [4, 12], potato [2], tobacco [9], rubber [6]. This paper is concerned with statistical point of view of the method.

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STATISTICAL MODEL

The usual procedure of the method involves measuring of lengths, widths, and areas of a sample of leaves and then calculating one of several possible correlation coefficients, regression coefficients or leaf factor for predicting areas of subsequent samples. The area is related to one or more linear measurements, which can be written as

$$Y = f(X) + \in \tag{1}$$

where Y = leaf area, X = linear measurement, f is unknown function, and $\in$ = error term. Generally adopted function is a linear one, and X is limited to one linear measurement only either leaf length, leaf width, or some derivatives of them. The model becomes.

$$Y = \alpha + \beta X + \in \tag{2}$$

or

$$Y = \beta X + \in \tag{3}$$

where β is leaf area factor. Model (3) seems more realistic, since leaf area should be zero if other dimension of leaf is zero.

ESTIMATION OF β

Let assume for the moment that

$$X \text{ is fixed} \tag{4}$$

$$\in \sim N(0, \sigma^2) \tag{5}$$

Case 1

Denote the estimate line by

$$\hat{Y} = \hat{\beta} x \tag{6}$$

where

$$\hat{\beta} = \Sigma XY / \Sigma X^2 \tag{7}$$

with variance

$$V\text{ar}(\hat{\beta}) = \sigma^2 / \Sigma X^2 \tag{8}$$

and the estimate of σ^2 is

$$\begin{aligned} \hat{\sigma}^2 &= \Sigma e^2 / (n-1) \\ &= [\Sigma Y^2 - \hat{\beta}\Sigma XY] / (n-1) \end{aligned} \tag{9}$$

It can be shown [3] that $\hat{\beta}$ is best unbiased estimate of β and σ^2 is an unbiased estimate of σ^2.

Case 2.

Other workers [1, 2, 10, 11, 13] estimated the line by

$$Y = a + bX \tag{10}$$

where

$$b = \Sigma xy / \Sigma x^2 \tag{11}$$

with variance

$$Var(b) = \sigma^2 / \Sigma x^2 \tag{12}$$

and the estimate of σ^2 is

$$s^2_{yx} = \Sigma e^2 / (n - 2)$$
$$= \frac{\Sigma y^2 - b\Sigma xy}{n - 2} \tag{13}$$

If model (3) is true, it can be shown that b and s^2_{yx} is an unbiased estimate of β and σ^2. However, it is clear from (12) and (8) that b has greater variance than $\hat{\beta}$.

From (11) $b = \Sigma xy / \Sigma x^2$

$$= \Sigma xY / \Sigma x^2$$
$$= \frac{\Sigma x (\beta X + \epsilon)}{\Sigma x^2}$$
$$= \beta + \Sigma x\epsilon / \Sigma x^2 \tag{14}$$

and the expectation is β under (5).

Averaging (3) over n samples gives

$$\overline{Y} = \beta\overline{X} + \overline{\epsilon} \tag{15}$$

Substracting (15) from (3)

$$Y - \overline{Y} = \beta(X - \overline{X}) + (\epsilon - \overline{\epsilon})$$
$$y = \beta x + (\epsilon - \overline{\epsilon}) \tag{16}$$

Define

$$e = Y - \hat{Y} \tag{17}$$
$$Y = \hat{Y} + e$$
$$= a + bX + e \tag{18}$$

Where b is given in (11) and a is

$$\overline{Y} - b\overline{X} \tag{19}$$

Substracting (19) from (18) gives

$$Y - \overline{Y} = b(X - \overline{X}) + e$$
$$y = bx + e$$
$$e = y - bx$$

Substituting (16)

$$e = -(b - \beta)x + (\epsilon - \overline{\epsilon})$$

Squaring and adding them up afterward comes to

$$\Sigma e^2 = (b - \beta)^2\Sigma x^2 + \Sigma(\epsilon - \overline{\epsilon})^2 - 2(b - \beta)\Sigma x(\epsilon - \overline{\epsilon})$$

and the expectation is

$$E(\Sigma e^2) = (n - 2)\sigma^2$$

Thus $\Sigma e^2 / (n - 2)$ is an unbiased estimate of σ^2.

Case 3.

Still other workers [5, 8] estimated β simply by computing leaf factor of each leaf from Y/X and then averaging over n samples :

$$\hat{\hat{\beta}} = (\Sigma Y/X)/n \tag{20}$$

with variance

$$s^2 = \frac{\Sigma(Y/X - \hat{\hat{\beta}})^2}{n - 1}$$

which after simplification becomes

$$s^2 = \frac{\Sigma(Y/X)^2 - (\Sigma Y/X)^2/n}{n - 1} \tag{21}$$

if model (3) is true, $\hat{\hat{\beta}}$ is also an unbiased estimate of β, as shown below :

$$\hat{\hat{\beta}} = \frac{1}{n} \Sigma\left[\frac{\beta X + \epsilon}{X}\right]$$

$$= \frac{1}{n} \Sigma\left[\beta + \frac{\epsilon}{X}\right]$$

$$= \beta + (\Sigma\epsilon/X)/n$$

with its expectation is β under (5). However, s^2 is greater than (8).

$$(n - 1)s^2 = \Sigma\left[\frac{\beta X + \epsilon}{X}\right]^2 - \left[\Sigma\left(\frac{\beta X + \epsilon}{X}\right)\right]^2 /n$$

$$= \Sigma(\beta + \epsilon/X)^2 - [\beta n + \Sigma(\epsilon/X)]^2/n$$

$$= \Sigma(\epsilon/X)^2 - (\Sigma\epsilon/X)^2/n$$

and its expectation

$$E[(n - 1)s^2] = [(n - 1)\sigma^2\Sigma(1/X^2)]/n$$

so

$$E(s^2) = [\sigma^2\Sigma(1/X^2)]/n$$

which is greater than (8).

CONCLUSION

It is more reasonable to treat leaf length or leaf width or their derivatives as a random variable, not a fixed one as assumed in (4). It has been shown [3] that for such a relaxation, all equations in deriving an estimate of regression coefficient and their related statistics are still hold. Therefore if the true model is $Y = \beta X + \in$, equation (7) gives the best estimate of leaf factor.

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