MODELLING OF NITROGEN x PLANT POPULATION INTERACTION ON CORN (ZEA MAYS L.)*)

Nasrullah **)

Summary

Two models, Model I and Model II, were proposed to describe the relationship of corn yield to the rate of nitrogen and plant population. The models were developed based on a slightly modified corn population-yield relationship of Duncan (1958) with the inclusion of the behaviour of the coefficients in relation to the rate of nitrogen. Model I has a consistently better fit than Model II, and a lower rank of mean square of lack of fit than ordinary and square root polynomial models. Model II has similar rank in comparison to the two polynomial models considered in the study.

Ringkasan

Hubungan antara produksi dan populasi tanaman telah banyak dimodelkan. Untuk tanaman jagung, model dari Duncan (1958) sangat tinggi tingkat keserasiannya. Dari model ini dapat ditentukan populasi tanaman yang optimum, yang akan memberi produksi yang maximum. Besarnya populasi tanaman yang optimum masih tergantung banyak faktor, tersedianya unsur hara terutama nitrogen adalah faktor terpenting.

Karangan ini mengetengahkan dua model, Model I dan Model II, untuk menggambarkan hubungan antara produksi jagung dengan pupuk nitrogen dan populasi tanaman. Kedua model tersebut dikembangkan dari model Duncan (1958) yang dimodifikasi dan kemudian dimasukkan hubungan antara koefisien-koefisiennya dengan pupuk nitrogen ke dalam model yang dimodifikasi. Model I selalu lebih baik tingkat keserasiannya dari Model II, dan mempunyai kwadrat-tengah-ketidak-serasian yang lebih rendah dalam urutan dengan model polynomial biasa dan polynomial transformasi akar.
Model II setara dengan kedua model polynomial tersebut.

Introduction

Corn yield, like other crops, is the product of the interaction between the plants and the environment in which they are grown. For a given variety planted in a certain area with a particular climatic conditions, the yield depends to a great extent on the number of plants per unit area. An excellent review on the quantitative relationship between plant population and crop yield is given by Willey and Heath (1969). Of various empirical models, the one originally proposed by Duncan (1958) and then modified by Carmer and Jackobs (1969) is of considerable fit for describing population-yield relationship in corn.

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* A digest of MS Thesis in Agronomy, University of the Philippines at Los Banos.

**) Department of Agronomy, Faculty of Agriculture, University of Gadjah Mada, Yogyakarta, Indonesia.

The optimum number of plants per unit area in turn depends on a number of factors, among which availability of plant nutrients is of considerable importance. When plants are widely spaced, they may be able to obtain from the relatively large volume of soil available to the individual plant an adequate supply of plant nutrients. However, the high potential yielding ability of maize variety can be achieved only, when soil moisture is not limiting, at plant density at which competition between plants for nutrients is relatively severe. Nutrients amount that would have been adequate for lower population become deficient and limiting, so that soil nutrients requirement increase. An adequate supply of nutrients is therefore essential under these conditions. Fertilizer nutrient most important for corn is nitrogen. Therefore increasing plant population should be accompanied by increasing amount of nitrogen applied to get high yield. Many investigations (Davide, 1962; Nandpuri, 1960; Singh, 1967; Sharma and Modgai, 1966; Sharma and Gupta, 1968; Colyer and Kroth, 1968; Gouda et al., 1976; Gouda and Bishr, 1976) supported the existence of the interaction of nitrogen and plant population.

A typical response of corn to plant population for different nitrogen levels was reported by Lang et al. (1956) who had conducted an experiment involving six plant populations on three levels of nitrogen availability. It was found that the higher the nitrogen available in the soil the greater the population that was required to obtain maximum yield.

Despite some other workers failed to show the interaction in their experiment (Vo, 1969; Lad and Khuspe, 1973), Earley (1967) included the interaction as a factor to be considered in boosting corn yield. If one is able to model the response of corn simultaniously to rate of nitrogen and plant population one will be able to exploit the interaction for maximizing corn yield. It is the aim of this study to develop such a model.

Model Development

Let Y, X_1, X_2 be corn yield, plant population in thousand per hectare and rate of nitrogen application.
The model proposed by Duncan (1958)

$$Y = X_1K(10)^{bX_1} \tag{1}$$

Where K and b are constants can be written as

$$Y = X_1Ke^{bX_1} \tag{2}$$

The optimum plant density is

$$X_1\text{opt} = -b^{-1} \tag{3}$$

Consider the application of (2) to relate yield per unit area to plant population for several rates of nitrogen application. Interaction of nitrogen and plant population implies that the optimum plant density is a function of the rate of nitrogen application

$$b = f(X_2) \tag{4}$$

Substituting (4) into (2), the proposed model is

$$Y = KX_1 e^{X_1 f(X_2)} \tag{5}$$

The form of $f(X_2)$ can be determined by examining the behaviour of b in relation to X_2 (Box and Hunter, 1962) or its transformation (Box and Tidwell, 1962).

The model was evaluated using the data of Colyer and Kroth (1968). Least square estimate of the coefficients in (5) were computed using iteration method of modified Gauss-Newton (Hartley, 1961) with the use of SAS program NLIN (Helwig and Council, 1979).

Second degree polynomial models, ordinary and square root transformation, were also fitted to the data for comparison of the fitness. Comparison was based on lack of fit of the models using Friedman test (Steel and Torrie, 1960; Evert and Howell, 1979).

Result and Discussion

Proposed Model

The estimates of the parameters in (2) are presented in Table 1 and Table 2. Examining Table 1 one will see no specific pattern of $K(X_2)$,K as a function of X_2, and it is not unreasonable to say that $K(X_2)$ does not depend on X_2, that is

$$K(X_2) = b_0 \tag{6}$$

where b_0 is a constant. However, one will notice from Table 2 that $b(X_2)$, b as a function of X_2, has a certain trend. With increasing rate of nitrogen the value of b increases sharply with a fairly abrupt transition to an almost constant value. The general tendency of $b(X_2)$ is depicted in Figure 1. Paralel figures were also observed for optimum plant density (Table 3). These are expected as the optimum plant density required is a function of b and is free from K as shown in (3).

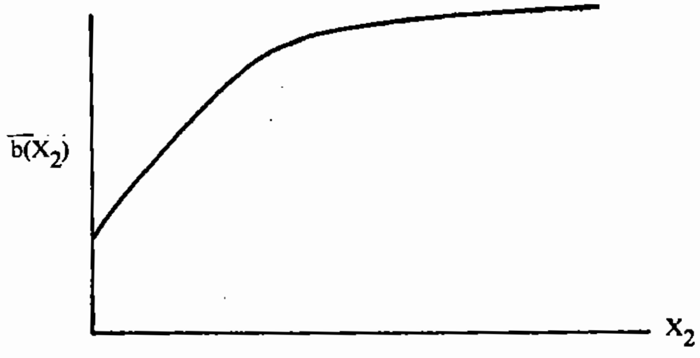

Figure 1. **The general tendency of b (X_2)**

The curve in Figure 1 can be represented in two mathematical forms :

$$b(X_2) = -b_1 - b_2e^{-b_3X_2} \tag{7}$$

or

$$b(X_2) = -b_1 + \frac{b_2b_3X_2}{b_2 + b_3X_2} \tag{8}$$

Substituting (6) and (7) to (2), and (6) and (8) to (2) give us two models, thereafter termed as Model I and Model II. Writing in full notation, Model I is

$$Y = b_0x_1\exp(-b_1 - b_2e^{-b_3X_2})X_1 \tag{9}$$

and Model II

$$Y = b_0X_1\exp(-b_1 \frac{b_2b_3X_2}{b_2 + b_3X_2})X_1 \tag{10}$$

If nitrogen does not interact with plant population, b_3 in both models tend to be very large. In this case (9) reduces to (2) since exp $(-b_3X_2)$ approaches zero. In Model II,for very large b_3,$b_2b_3X_2/(b_2 + b_3X_2)$ tend to b_2.So by equating $b = -b_1 + b_2$, equation (10) reduces to (2). The numerical value of b_3 can therefore be used as an indicator of the magnitude of the interaction of nitrogen and plant population, and can appropriately be termed as an index of the interaction. The greater the value of b_3 the smaller the interaction.

Table 1. **Least square estimate of K in equation (2) for different rates of nitrogen.**

Location	Year	Nitrogen (kg/ha)						
		0	28	56	84	112	168	224
Seymour	1961	5.2832	3.9637	3.2213	3.2731	3.4419	3.2137	3.2251
	1962	3.5189	4.0693	4.0511	4.0172	4.1087	4.3958	3.8717
	1963	5.7717	4.8726	5.2910	4.6341	5.1164	5.1536	4.5730
	1964	8.0837	5.1780	7.7654	6.8739	7.3683	7.5843	6.5295
	1965	5.4508	5.5259	6.5748	5.0168	6.6616	5.7372	5.6173
	1966	5.5463	5.4906	6.3433	6.6089	6.5848	6.6000	6.1279
	1967	3.8440	3.4688	4.4155	4.3460	5.3344	5.6822	5.5866
Marshall	1961	3.9457	4.0479	4.2388	3.8498	4.6894	4.1706	3.3742
	1962	5.7158	6.0023	5.1989	5.4220	5.2024	5.7491	6.0209
	1963	4.7391	4.3379	4.4898	4.4949	4.0494	4.5380	4.5033
	1964	6.8382	5.7047	5.8965	5.8785	5.5746	5.6264	5.8879
	1965	6.7428	6.2411	5.7141	5.4935	5.3722	5.5232	5.2753
	1966	5.3617	4.9429	5.0277	5.0265	4.6197	4.7962	4.4513
	1967	6.9719	7.8656	5.3918	5.5039	4.7980	4.5180	5.0625

Table 2. **Least square estimate of b in equation (2) for different rates of nitrogen.**

Location	Year	Nitrogen (kg/ha)						
		0	28	56	84	112	168	224
Seymour	1961	-.0406	-.0296	-.0211	-.0191	-.0197	-.0166	-.0173
	1962	-.0372	-.0336	-.0272	-.0212	-.0210	-.0204	-.0173
	1963	-.0408	-.0336	-.0337	-.0250	-.0273	-.0224	-.0179
	1964	-.0557	-.0371	-.0403	-.0349	-.0378	-.0360	-.0326
	1965	-.0418	-.0370	-.0364	-.0266	-.0318	-.0259	-.0255
	1966	-.0392	-.0320	-.0306	-.0287	-.0289	-.0274	-.0253
	1967	-.0384	-.0325	-.0367	-.0304	-.0338	-.0281	-.0253
Marshall	1961	-.0291	-.0257	-.0247	-.0213	-.0257	-.0216	-.0158
	1962	-.0346	-.0329	-.0288	-.0307	-.0294	-.0324	.-0335
	1963	-.0367	-.0332	-.0339	-.0338	-.0302	-.0350	-.0345
	1964	-.0374	-.0287	-.0289	.-0286	-.0274	-.0272	—.0293
	1965	-.0376	-.0297	-.0245	-.0218	-.0206	-.0215	-.0205
	1966	-.0345	-.0273	-.0253	-.0247	-.0223	-.0228	-.0219
	1967	-.0515	-.0488	-.0313	-.0261	-.0215	-.0199	-.0230

Taking the first derivative of (9) or (10) with respect to X_1 and equating it to zero, one gets after simplification.

$$X_1\text{opt} = \frac{1}{b(X_2)} \tag{11}$$

which is the inverse of (7) or (8). Assuming that the relationship between yield, rate of nitrogen application and plant density can be appropriately described by Model I or Model II, the above equation gives the relationship of the optimum plant density required to get maximum yield for any nitrogen level. Thus one will be able to maximize yield on a limited input, either seed or nitrogen. For unlimited source of seed and nitrogen, the proposed models unfortunately do not provide the optimum solution to give the maximum yield possible. The method of Walker and Carmer (1967) might be used in this case. It is not considered here because of the complexity involved.

Table 3. **Optimum plant density for different rates of nitrogen based on equation (2).**

Location	Year	Nitrogen (kg/ha)						
		0	28	56	84	112	168	224
Seymour	1961	24,660	33,836	47,383	52,448	50,641	60,350	57,820
	1962	26,853	29,784	36,380	47,077	47,550	48,966	57,671
	1963	24,530	29,758	29,639	39,939	36,626	44,575	55,969
	1964	17,940	26,951	24,800	28,659	26,449	27,786	30,703
	1965	23,937	27,020	27,436	37,654	31,401	38,559	39,146
	1966	25,527	31,279	32,652	34,787	34,624	36,517	39,515
	1967	26,033	30,751	27,233	32,867	29,608	35,526	39,592
Marshall	1961	34,311	38,381	40,464	47,042	38,931	46,202	63,414
	1962	28,868	30,350	34,743	32,586	33,962	30,828	29.829
	1963	27,229	30,161	29,469	29,598	33,087	28,587	28,988
	1964	26,715	34,904	34,636	34,972	36,503	36,717	34,186
	1965	26,627	33,644	40,859	45,885	48,590	46,434	48,891
	1966	28,950	36,608	39,451	40,505	44,798	43,805	45,676
	1967	19,410	20,494	31,979	38,386	46,447	50,178	43,491

Table 4. **Least square estimates of the parameters and mean squares of lack of fit of Model I.**

Location	Year	b_0	b_1	b_2	b_3	MSE Model	Pooled Var	F-test	Prob
Seymour	1961	3.5146	.0187	.0113	1.6806E-02	16.25	15.70	1.04	.49
	1962	4.0562	.0177	.0250	1.9022E-02	9.85	9.77	1.01	.51
	1963	4.9766	.0127	.0247	0.5647E-02	18.59	21.35	.87	.63
	1964	7.0092	.0342	.0185	2.5320E-02	17.04	14.64	1.16	.39
	1965	5.8095	.0253	.0188	1.5656E-02	9.45	8.61	1.10	.44
	1966	6.2847	.0258	.0170	2.3157-02	10.50	11.03	.95	.56
	1967	4.9947	.0083	.0387	0.4637E-02	19.73	15.97	1.24	.35
Marshall	1961	4.0292	.0206	.0091	2.1235E-02	2.88	2.35	1.23	.35
	1962	5.6059	.0314	.0028	1.7643E+17	3.18	2.58	1.23	.35
	1963	4.4410	.0337	.0014	8.9322E-02	3.25	3.46	.94	.57
	1964	5.8772	.0288	.0043	6.6077E-02	3.93	4.47	.88	.62
	1965	5.6534	.0218	.0114	2.8729E-02	9.83	10.01	.98	.53
	1966	4.8485	.0234	.0087	3.5302E-02	5.68	6.92	.82	.68
	1967	5.2528	.0226	.0251	2.2434E-02	35.84	19.00	1.89	.11

Table 5. **Least square estimates of the parameters and mean squares of lack of fit of Model II.**

Location	Year	b_0	b_1	b_2	b_3	MSE Model	Pooled Var	F-test	Prob
Seymour	1961	3.5147	.0299	.0143	2.3515E-04	17.14	15.70	1.09	.44
	1962	4.0528	.0427	.0305	6.4239E-04	13.85	9.77	1.42	.25
	1963	4.9760	.0374	.0394	1.4463E-04	18.89	21.35	.88	.62
	1964	6.9974	.0526	.0217	6.7305E-04	19.94	14.64	1.36	.28
	1965	5.8069	.0442	.0240	3.7363E-04	11.74	8.61	1.36	.28
	1966	6.2789	.0430	.0203	5.6904E-04	13.04	11.03	1.18	.38
	1967	4.9930	.0470	.0641	1.8285E-04	20.08	15.97	1.26	.33
Marshall	1961	4.0278	.0298	.0111	2.7559E-04	3.10	2.35	1.32	.30
	1962	5.6059	.0341	.0028	5.6835+16	3.31	2.58	1.28	.32
	1963	4.4411	.0350	.0013	1.0000E+30	3.26	3.46	.94	.57
	1964	5.8770	.0331	.0045	8.8665E-04	4.07	4.47	.91	.59
	1965	5.6532	.0333	.0131	5.0988E-04	12.99	10.01	1.30	.31
	1966	4.8472	.0321	.0097	5.2874E-04	6.86	6.92	.99	.52
	1967	5.2533	.0469	.0291	7.3853E-04	51.10	19.00	2.69	.03

Goodness of Fit of the Models

Table 4 and Table 5 show the least square estimates of the parameters of Model I and Model II. The seventh column is mean squares of lack of fit of the corresponding model and the eighth column is the pooled mean squares of lack of fit upon fitting (2) individually to the data. The last column shows the probability of rejection on testing the equivalence between the two estimates of mean square of lack of fit. For Model I, one can conclude that the inclusion of X_2 into (2) through (7) has not caused any degradation in the quality of fit (Steel and Torrie, 1960; Scoot and Sylvestre, 1979).

Table 6. **Mean squares of lack of fit of the models (upper part) and their corresponding ranks (lower part).**

Location	Year	Model I	Model II	2 nd Degree Ordinary	Polynomial Square-root
Seymour	1961	16.25	17.14	9.42	7.59
	1962	9.85	13.85	12.23	22.66
	1963	18.59	18.89	8.01	11.42
	1964	17.04	19.94	24.29	25.60
	1965	9.45	11.74	10.08	20.26
	1966	10.50	13.04	13.85	17.19
	1967	19.73	20.08	21.26	20.28
Marshall	1961	2.28	3.10	2.90	3.48
	1962	3.18	3.31	7.06	4.36
	1963	3.25	3.26	2.77	2.55
	1964	3.93	4.07	7.23	3.42
	1965	9.83	12.99	19.77	11.29
	1966	5.68	6.86	9.84	4.71
	1967	35.84	51.10	41.20	71.96
Seymour	1961	3	4	2	1
	1962	1	3	2	4
	1963	3	4	1	2
	1964	1	2	3	4
	1965	1	3	2	4
	1966	1	2	3	4
	1967	1	2	4	3
Marshall	1961	1	3	2	4
	1962	1	2	4	3
	1963	3	4	2	1
	1964	2	3	4	1
	1965	1	3	4	2
	1966	2	3	4	1
	1967	1	3	2	4
Total Rank		22	41	39	38

$$X^2 = \frac{12}{bt\,(t+1)} \Sigma r_i^2 - 3b\,(t+1)$$

$$= 9.857 \text{ with } 3 \text{ d.f.}$$

For Model II, except for 1967 experiment in Marshall, the general tendency remains the same. Model I, however, is better than Model II since the mean square of lack of fit of Model I are consistently smaller than that of Model II.

Model Comparison

Table 6 shows the mean squares of lack of fit of the two models in addition to those of second degree polynomial models commonly used in practice, ordinary and square-root transformation. To avoid complication in their comparison using appropriate parametric statistical test, a nonparametric statistical test developed by Friedman (Steel and Torrie, 1960; Evert and Howell, 1979) was used. Result of the test showed that the models have different rank of mean squares of lack of fit, and Model I has the smallest rank; the other three models rank equally to each other.

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