THE EFFECT OF ASYMMETRICAL INFORMATION AND RISK ATTITUDE ON INCENTIVE SCHEMES: A CONTINGENCY APPROACH

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ABSTRACT

This paper discusses several incentive models. Some models are only appropriate for risk neutral agents, but not for risk averse agents. For certain circumstances, a quadratic model is needed to replace a linear model. The choice of which model is the appropriate one is the question addressed by this paper. Risk attitude and several aspects of asymmetric information will be discussed.

INTRODUCTION

The relationship between principals and agents commonly involves a situation of asymmetrical information. Agents have more information about their own capacity and the working environment than principals do. Assuming that individuals act to maximize their own benefit, the presence of asymmetric information will motivate agents to conceal any information unknown to principals so far as it can promote the agents' self-interest. This agency issue is known as hidden information and hidden action problems.

In a participatory budget setting situation, the hidden information problem suggests that the agents may not truthfully reveal his private information. In other words, the agents create slack by setting an easy budget that is knowingly understate their expected performance (Schiff and Lewin 1970). In particular, when the agents' performance is evaluated on the basis of their budget targets, the agents will be motivated to engage in this slack-creating activity. As Young et al. (1988, pp. 111-
argued, "When a subordinate's pay increases as budget difficulty decreases, ceteris paribus, he or she may bias the communication of private information such that a relatively easy budget is set..." The tendency of the agents to engage in this slack-creating activities reduces the potential benefits of budget participation systems as controlling and motivational devices. Budget targets do not 'truly' represent employees' maximum efforts. Accordingly, principals' attempt to optimize the organization's resources are inhibited.

Compensation schemes greatly affect the extent to which this slack-creating problem can be mitigated. The principals' main task is, therefore, designing an effective compensation scheme. At present, there are several compensation schemes available in the literature. These compensation schemes, however, have very strict assumptions which, quite often, do not reflect the real condition. Their applicability, therefore, is very limited. For example, it is assumed that agents are risk-neutral but empirical evidence demonstrated that some agents were risk averse (Chow et al. 1988; Waller 1988) Furthermore, it is (implicitly) assumed that agents' actions are observable. In most cases, however, agents actions are unobservable.

This paper attempts to expand the applicability of the compensation models by relaxing these two strict assumptions. In addition, it attempts to articulate conditions which are appropriate for a certain compensation scheme. That is, the paper also attempts to develop a contingency model of compensation schemes. It is hoped that this paper will be able to help managers in designing appropriate incentive schemes for their organization.

The rest of the paper is organized as follows. Section 2 discusses a linear incentive scheme, the Soviet models (Picard 1987; Kirby et al. 1991) Section 3 discusses the extended model with several assumptions relaxed: agents are treated as risk-neutral individuals and agents' actions are assumed to be unobservable. Section 4 discusses the quadratic incentive scheme which will be followed by Conclusion in Section 5.
THE LINEAR INCENTIVE MODELS

The popular linear incentive models are the Soviet incentive model, Picard model and Kirby et al. model. Basically, these three models are similar. The models can be described as follows.

\[
B = B' + \beta (Y'' - Y') + \alpha (Y - Y''), \quad \text{if } Y \geq Y''
\]

\[
B = B' + \beta (Y'' - Y') + \tau (Y - Y''), \quad \text{if } Y < Y''
\]

where, \(B'\) and \(Y'\) are the initial bonus and budget levels respectively; \(Y''\) is the participatively-set budget; \(Y\) is the actual performance; and \(\alpha, \beta\) and \(\tau\) are the reward/penalty coefficients.

These models have three stages Weitzman, 1976, p. 2531 First, the preliminary stage, where the principal sets \(B'\) and \(Y'\) as the initial bonus and initial budget levels, respectively, to meet the quota. The second or planning stage is actually the participation stage, in which agent has a chance to determine a larger or a smaller planned target \(Y''\), which correspondingly determines a larger or smaller planned bonus \(B'+ \beta (Y''- Y')\)- By setting a higher planned target, \(Y''\), than the initial budget level, \(Y'\), agents have an incentive to obtain an additional bonus \(b (Y''- Y')\)- The third stage, the implementation stage, is related to the actual bonus received for actual performance \(Y\). If actual performance \(Y\) greater (equal) to targeted performance \(Y''\), actual bonus received will be greater (equal) than planned bonus. If \(Y < Y''\), actual bonus received will be smaller than planned bonus, because of penalty for unable to meet the quota. This penalty is necessary to discourage agents from setting a reasonably high plan target in the budget setting stage.

The reward/penalty coefficients are closely related. The extent to which agents are willing to reveal their private information is determined by how the principals determine the rewards/penalty. These coefficients should be determined in such a way so that \(0 < \alpha < \beta < \tau\). That is, to induce agents to report their private information truthfully, principals ought to determine the magnitude of the reward/penalty in this order. First, when \(\alpha < \beta\), the agents would not get higher bonus by biasing \(Y''\) downward and then having favorable performance. Second, when \(\beta < \tau\), the agent
would not get higher bonus by biasing $Y''$ upward and then having unfavorable performance. Eventually, because $\alpha > 0$, the agent has an incentive to have higher performance. 

There is always an incentive to be truthful in a condition of certainty. But, for an uncertain condition, it will be optimal for a risk-neutral agent to set $Y''$ to maximize the expected bonus if the following expression holds.

$$P(Y \geq Y'') = \frac{\tau - \beta}{\tau - \alpha}$$

(2)

Where $P(Y \geq Y'')$ is $\int_{y} f(Y)dy$. 

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1 Technical example of using this scheme and its related coefficients are described in Mann (1988).

3 The numerical example is given in Waller (1988, p. 89).

The derivation of this expression is not detailly given in Wetizman (1976). The detail one is as follows.

Suppose that the probability density function of $Y$ is $f(Y)$ and $\int f(Y)dy = 1$.

Max $B = \int_\cdots \left[ B' + \beta(Y'' - Y') + \alpha(Y - Y'') \right] f(Y)dy + \int \left[ B' + \beta(Y'' - Y') + \tau(Y - Y'') \right] f(Y)dy$

$Y''$

Differentiate with respect to $Y''$:

$\int (\beta - \tau) f(Y)dy + \int (\beta - \alpha) f(Y)dy = 0$

$\int (\tau - \beta) f(Y)dy = \int (\beta - \alpha) f(Y)dy$

$\int f(Y)dy = \left[ (\tau - \beta)/(\beta - \alpha) \right] f(Y)dy$

$\int f(Y)dy = \left[ (\tau - \beta)/(\beta - \alpha) \right] \{ 1 - \int f(Y)dy \}$

$\int f(Y)dy = \left[ (\tau - \beta)/(\beta - \alpha) \right] - \left[ (\tau - \beta)/(\beta - \alpha) \right] \int f(Y)dy$

$\int f(Y)dy + \left[ (\tau - \beta)/(\beta - \alpha) \right] f(Y)dy = \left[ (\tau - \beta)/(\beta - \alpha) \right]$
A similar linear model with the Soviet incentive scheme was developed by Picard [1987] based on Laffont and Tirole's work [1986]. In his model, Picard [1987] jointly employs moral hazard and adverse selection features. Moral hazard results from hidden action, while adverse selection exists because of hidden information problem.

Suppose that $\infty$ is a cost parameter, $a(\infty)$ is level of output, $\Phi(a(\infty))$ is disutility of effort, and $t(\infty, x)$ is the incentive. Principal’s outcome, $x$ is $a(\infty) - \infty + e$, where $e$ is a random variable $(0, \sigma^2)$ with density function $g(e)$. Let $r(\infty) = a(\infty) - \infty$, Picard model for optimal incentive from principal viewpoint then can be formulated as follows.\(^4\)

$$\int_{\Phi}^{\Phi_1} t(\infty, x) = \Phi(a(\infty)) + \int \Phi'(a(s)) ds + \Phi'(a(\infty))(x - r(\infty))$$

This scheme is similar with that of the Soviet incentive model, where $\Phi(a(\infty)) + \int \Phi'(a(s)) ds$ is a fixed initial bonus equal to $B' + \beta(Y'' - Y')$, and $\Phi(a(\infty))(x - r(\infty))$ is additional bonus equal to $a(Y - Y'')$ or penalty equal to $\tau(Y'' - Y)$ for those in the Soviet incentive scheme.

A similar but clearer model is presented by Kirby et al. (1991) While other models adopt an output maximization problem, Kirby's et al. model adopts a cost

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4 We argue that his model actually does not include hidden action problem. I will discuss it in section 3.
minimization problem. Suppose EC is estimated cost reported by the agent, AC is the actual cost, and v(EC) and w(EC) are positive value functions of expected cost, then their incentive scheme can be written as: 

\[ H(EC, AC) = v(EC) + w(EC) \cdot (EC - AC) \]  

Here again, this scheme is exactly the same as the Soviet incentive model, where v(EC) is the initial bonus, w(EC) (EC - AC) is additional bonus/penalty depends on favorability of the budget variance (EC - AC).

**THE EXTENDED MODEL**

As noted in the Introduction, the extended model incorporates the issues of risk aversion and hidden action problems. The previous models assume that both the principals and the agents are risk neutral. Because both principals and agents are risk-neutral, their utilities are the same with their expected returns. This means that the ordinal value of the agents' utility and the ordinal value of the principals' utility are exactly the same. As a consequence, the principal welfare function can have the agents' utility function as the functional argument (see footnote number 5 for this discussion). However, if principals are risk-neutral and agents are risk-averse, the agents' and the principals' ordinal values of the utility are not the same. In other words, agents' utility cannot be substituted directly into principals' welfare function. Accordingly, the previous models are not applicable in a situation where principals are risk neutral but agents are risk-averse.

In addition, empirical evidence also demonstrates that risk preference is an important factor in incentive scheme [Young 1985; Waller 1988] Risk-neutral agents will response differently than risk-averse agents to incentive alternatives offered by principal. This suggests that separate incentive schemes for risk-neutral agents and risk-averse agents are needed.

The previous models address the unobservability of agents' actions (hidden action problem) by measuring the agents' output. Picard [1987] for example, instead of addressing the agents' actions, included the agents' level of output in his model. This approach, however, creates an adverse selection problem. It is argued herein that
to really incorporate hidden-action problem, the model should consider the worse alternative action the agent will do if the incentive he would receive from the best action is not persuasive. This contraint will be included in the extended model.

**The Basic Framework**

The extended model is developed based on Holmstrom [1979], Harris et al. [1982], Picard (1987) and Varian's [1992] work. Following Varian's [1992], it is assumed that there are n possible output levels (X₁, ..., Xₙ). Furthermore, given that agents' actions cannot be observed directly, there is a probability that an action would be taken. The probability of an agent to take the best action (b) to produce output level Xi is Pib and the probability of the agent to take an alternative action a is Pia.⁵

Picard [1987] asserts that the principal welfare is output level minus costs of producing these output minus incentives paid to the agent. Using this framework, the principal welfare if the agent performs action (b) is:

\[
W = \sum_{i=1}^{n} \left[ \left( X_i(\infty) - \infty - I(X_i(\infty)) \right) \cdot Pib \right]
\]

where \( X_i = \) output level
\( \infty = \) cost parameter
\( I = \) incentive paid to agent
\( Pib = \) probability of agent taking the best action b.

As a risk-averse individual, the agent will maximize his von Neumann-Morgenstern utility function of the incentive she or he will receive. Suppose the agent's costs of action j is Cj. It is necessary to set a constraint to induce agent to take the best action. However, the principal does not know this best action.

⁵ In Holmstrom’s (1979) model, the agent only produces two kind of outputs, X₁ represents state of world 1 as the bad state, and output X₂ represents state of world 2 as produce X₁ is P and the probability of agent to produce X₂ is (P-1), if I₁ and I₂ are the incentive for producing X₁ and X₂ respectively, so principal expected utility can be formulated as \( P \times U(X₁ - I₁) + (1-P) \times U(X₂ - I₂) \).
Accordingly, what the principal can do is to influence the action by giving incentives at least as big as if the agent is taking another alternative action. This constraint is called *self-selection condition* [Harris et al. 1982] or *incentive compatibility constraint* [Varian, 1992] which can be written as follows.

Agents also have another option not to participate. That is, the agents may not take either action (a) or (b). In other words, the agents may have another opportunity (for example, they are working with another principal) that yields a certain reservation of utility. Suppose, if agents do not participate, they will get utility U. Hence, by participating, the agents have to have utility at least the same as U. To enforce agents not to work with other principals, another constraint is needed. This second constraint is called the individual rationality constraint (Harris et al. 1982) or the participatory constraint (Varian 1992). It can be expressed as follows:

\[
\sum_{i=1}^{n} U(I(X_i(\pi))) \frac{P_i}{P_b} - C_b \geq U
\]  

(7)

**Optimality and Linearity of the Scheme**

The Lagrangian for solving the maximization problem can be written as:

\[
\lambda = \sum_{i=1}^{n} [(X_i(\pi) - \Phi - I(X_i(\infty)) \cdot P_i \] 

- \lambda [C_b - \sum_{i=1}^{n} U(I(X_i(\pi))) (P_i - P_a)] 

- \mu [C_b - U \sum_{i=1}^{n} U(I(X_i(\pi))) P_i] 

(8)

Differentiate with respect to I:

\[-P_b + \sum_{i=1}^{n} \frac{U(I(X_i(\pi))) (P_b - P_a) + \mu U(I(X_i(\pi))) P_b}{U'(I(X_i(\pi)))} = 0\]

Divided by \((P_b) U'(I(X_i(\pi)))\) we get:

\[
\frac{1}{U'(I(X_i(\pi)))} = \mu + \lambda \left[ \frac{P_a}{P_b} \right]
\]

(9)

\(\frac{P_a}{P_b}\) is a likelihood ratio. It measures the ratio of likelihood that agent chooses action a to the action b. The greater the ratio, the likelihood to the agent to choose action a.

The optimal result in equation (9) suggests that incentive payment consists of fixed bonus (n) and variable bonus (the part after the plus sign). The greater the probability of agent choosing the best action b, the greater the variable bonus will be. The fixed bonus relates to the participation constraint and the variable bonus relates to the incentive compatibility constraint. This incentive scheme is a linear function to the likelihood ratio.
Observability of Action

It is argued in his paper that incentive compatibility constraint is necessary if there is a hidden action problem (agents' action cannot be observed). This leads to the proposition as follows.

Proposition 1. If agent's action can be observed by the principal, the incentive compatibility constraint is inessential. Proof. Because agent's action now can be observed, incentive payment will be a function of the action rather than the output directly. The agent will get some certain incentive payment $I(X(b))$ or $I(X(a))$ for performing action $a$ or $b$ respectively to produce output $X$. The incentive payment is certain, so that in the constraints, probability $p_{ia}$ and $p_{ib}$ are not needed again as follows.

\[
U(I(X(b)) - C_b \geq U(I(X(a)) - C_a)
\]

(incentive compatibility constraint)

\[
U(I(X(b)) - C_b \geq U(I(X(a)) - C_a)
\]

(participation constraint)

Because the principal now knows that action $b$ is the best, he can set up an incentive that force agent to choose action $b$. This incentive can be $U + C_b$ if agent chooses action $b$, otherwise if agent chooses action other than $b$. This is clear that participation constraint is binding and incentive compatibility constraint is inessential. This incentive system is called target output scheme, a target output $X$ is set and agent will get incentive at his reservation price if he reaches this target, otherwise he will get a punishment [Varian 1992].

Proposition 2. When action is unobservable and the incentive compatibility constraint is binding, any change that makes the agent better off will make the principal worse off. Proof.

Differentiate equation (8) with respect to $p_{ia}$ we get:

\[
\frac{\delta U}{\delta p_{ia}} = \frac{\lambda}{n} \sum_{i=1}^{n} U(I(X_i(x)))
\]
From the *envelope theorem* \( \frac{\delta?}{\delta \text{Pia}} \) means the effect of \text{Pia} against the optimal value function. In this case, if the *incentive compatibility constraint* is binding \( \lambda > 0 \), the change of probability alternate action is opposed to the principal optimal value. Any change that makes the agent better off will make the principal worse off.

The degree of risk aversion varies from individual to individual. Varian (1992, p.454) gives an equation as follows:

\[
\tau = \frac{1}{1 + r c'(\alpha)}
\]

If \( \sigma^2 \) is equal to zero, it means there is no risk and \( \tau \) will be equal to 1. The greater the risk or the greater the absolute risk aversion, the smaller the value of \( \tau \). The absolute risk aversion is a measure of the degree of individual risk aversion. This value can be measured using the Arrow-Pratt measure as \( \frac{U''}{U'} \), where \( U' \) and \( U'' \) are the first and the second derivative of the utility function.

**THE QUADRATIC INCENTIVE SCHEMES**

Picard [1987] shows that optimal incentive payment can always be (approximately) formulated using quadratic incentive scheme. This scheme is robust to error and the disturbance of random variable. The quadratic scheme for principal's optimal strategy given by Picard [1987] is as follows.\(^6\)

\[
\tau (\infty, x) = (H/2) ((X-\tau(\infty))^2 - \sigma^2) + \Phi'(a(\infty)) (X-\tau(\infty)) + \Phi (a(\infty)) + \int \Phi'(a(s))ds.
\]

Kirby et al. [1991] also demonstrate that if the separation of cost environments (low cost and high cost environments) is not optimal, the linear incentive scheme could fail and the optimal one is the quadratic scheme. They then reformulated their model as follows.

\[
H(BC, AC) = V(BC) + W(BC) (BC-AC) - q [(AC-BC)^2 \sigma^2].
\]

\(^6\) the derivation of this equation is provided by Picard (1987) in his appendix.
This scheme is exactly the same as that of Picard in equation (11), where $V(EC) = \Phi(a(\infty)) + \int \Phi'(a(s)) \, ds$, $W(EC) = \Phi'(a(\infty))$, $-q$ is $H/2$, $EC$ is $X$ and $Ac$ is $r(\infty)$.

**CONCLUSION**

Several types of incentive scheme have been discussed. These are grouped into linear versus quadratic incentive schemes, agent's risk-neutral versus agent's risk-averse incentive schemes, observable versus unobservable action incentive schemes. Figure 1 below classifies the incentive scheme models into these categories.

The extended model is not intended to replace the other models. The extended model is as a complement to the others. The choice of the model for specific situation is important. One model is not always appropriate for every situation. The specific model is contingent on the specific situation. This *contingency approach* is consistent with the empirical evidence. Waller's [1988] study for example, concluded that an erroneous assumption about the agent's risk preference may impair the benefit of the scheme's information. This means that using an agent's risk-neutral scheme (such as the Soviet incentive scheme) to a risk-averse agents is incorrect application. For risk-averse agents, we must use an agent's risk-averse scheme in order to be effective in applying optimal incentive system. Another example is outlined by Kirby et al. [1991] in the choice of linear or quadratic scheme. They suggest that if the separation of cost environments is optimal, linear scheme is not a wrong choice, otherwise the optimal model is the quadratic scheme.
REFERENCES