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Direct Position Estimation (DPE): A Potential Application in Geodetic Networks

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ABSTRACT

Direct Position Estimation (DPE) is an emerging technique in the Global Navigation Satellite System (GNSS) used to estimate a user's position, velocity, and time directly from the correlation values of the received GNSS signals with the receiver's internal replica signals. Unlike the traditional two-step (2SP) approach, DPE determines the position directly from the sampled data without requiring intermediate steps. It achieves this by combining signal tracking with navigation techniques that compare the expected signal reception of multiple potential navigation candidates against the actual received signal. Theoretical studies suggest that DPE-based GNSS receivers can provide more robust localization compared to conventional 2SP receivers. Algorithms for DPE localization, which compute the navigation solution directly within the navigation domain, have been proposed to tackle the challenges faced by traditional 2SP receivers, albeit at the cost of increased computational load. Despite this higher computational requirement, DPE is a more effective positioning algorithm regarding multipath mitigation. The technique's resilience against multipath effects and non-line-of-sight (NLOS) conditions could also make it suitable for applications in geodetic networks, where robust estimators are typically utilized to counteract outliers.

Keywords: Direct Position Estimation (DPE), Geodetic Networks, Robust Estimator

1. Introduction

GNSS can provide users with continuous and real-time spatial location information on a global scale. It has outstanding advantages such as wide coverage, high precision, and all-weather. It has been widely used in both civilian and military fields. With the continuous expansion of positioning and timing applications, navigation receivers are also facing more and more complex working environments, which is a serious challenge to navigation signal-receiving technology.

On the navigation receiver side, most of the currently used scalar receivers are of the scalar tracking loop (Dou J. et.al, 2021 & Weill, 2010) its architecture is as shown in Figure 1. In a scalar tracking loop receiver, a satellite is tracked through each channel, and there is no information exchange between channels. The discriminator in each channel calculates the pseudo-code and carrier phase signals from the satellites respectively, and then it passes through the loop filter and directly feeds back to the carrier/code Numerically Controlled Oscillator (NCO) of this channel. NCO adjusts the frequency of the local pseudo-code and carrier phase with the local replica signals, which are consistent with the code phase and carrier frequency of the received signal. Thereby correlating the pseudo-code and carrier of the received signal with the local replica signal, could result in time delay and phase difference, which are the basic measurement parameters of positioning and timing determination. The satellite's orbits which are modulated in the navigation signal are also received and then despreaded as the navigation data.

This architecture is simple and easy to implement, but it has the following shortcomings:

i. Poor tracking sensitivity and dynamic performance (Sun W. et.al, 2021).
   In weak signal environments or highly dynamic scenarios, the tracking error of the tracking loop increases sharply. This makes it impossible to maintain stable tracking of satellite signals and perform accurate positioning and timing calculations.
ii. Poor tracking continuity and signal availability.
   Tracking errors can occur due to loss of signal lock or temporary obstructions, which can lead to scattered data that cannot be maintained within a narrow range. When the satellite signal strength returns to normal, it reacquires scalar tracking (Yang Y. et.al, 2021).

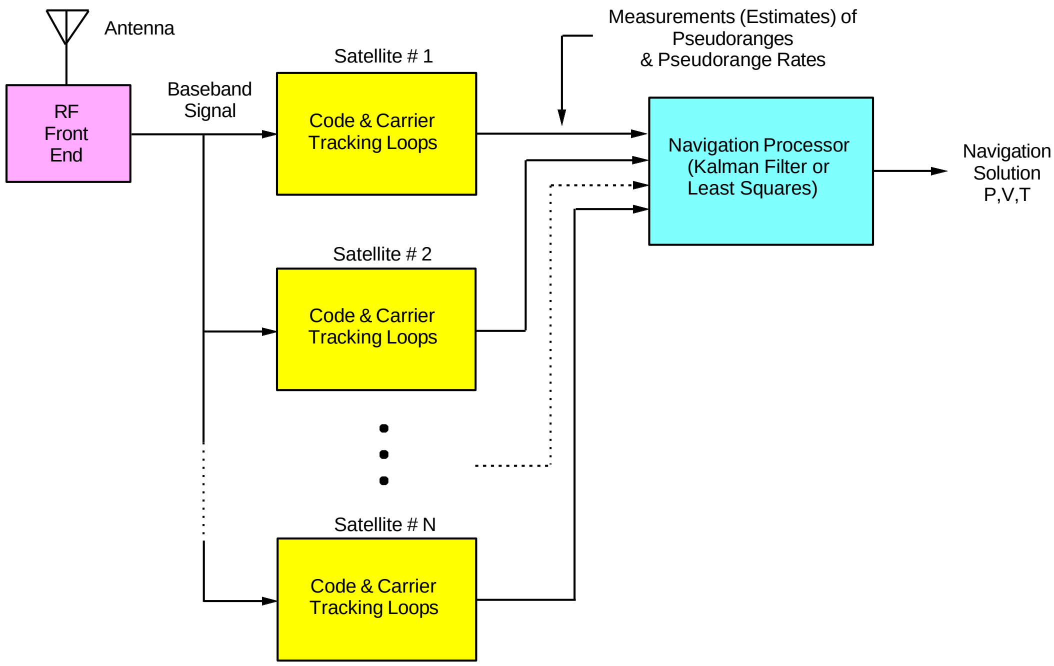

Figure 1. Conventional Scalar GNSS Signal Tracking Architecture. (*Source:* Reproduced with ION permission)

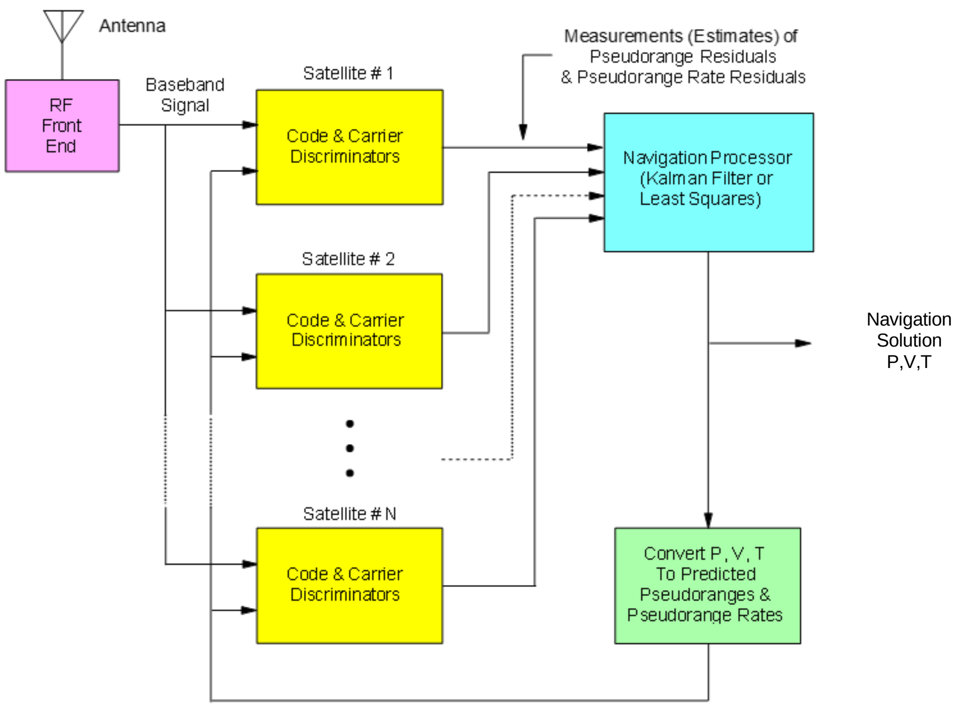

Figure 2. Typical Vector Tracking Loop. (*Source:* Reproduced with ION permission)

iii. The anti-interference performance is poor.

In a scalar tracking loop, each tracking channel operates independently. If one channel experiences interference, the tracking error for that channel will increase (Chang J., et.al, 2021). Unfortunately, the other channels cannot help track it, which may ultimately result in a loss of lock.

On the other hand, the vector receivers have made architectural improvements in response to the above three shortcomings. The control variables are no longer provided by individual loop filters but are generated uniformly by the navigation filter. Tracking and navigation solutions are integrated to achieve joint processing of multi-channel signals as shown in Figure 2. This allows the vector receiver to be improved in terms of tracking sensitivity, dynamic performance, anti-interference performance, etc., and is also suitable for signal occlusion scenarios. No need to recapture Vector receivers have made significant architectural improvements to address the three shortcomings mentioned earlier. Instead of relying on individual loop filters for control variables, these variables are now generated uniformly by the navigation filter. This integration allows for the joint processing of multi-channel signals, as shown in Figure 2. As a result, vector receivers exhibit enhanced tracking sensitivity, dynamic performance, and anti-interference capabilities. They are also well-suited for scenarios involving signal occlusion, eliminating the need for signal recapture (Cai, N., et.al, 2017)

However, vector tracking receivers also have some shortcomings, including:

i. The estimation of the code phase by the vector tracking algorithm is biased, and its convergence point is neither unique nor zero (Xiao Z., et.al, 2010).
ii. Because of the mutual coupling between channels, an error in one channel affects the others.
iii. The vector tracking loop requires significant computational resources and is costly to implement..

Although vector tracking helps address some of the limitations of scalar tracking to a certain extent, it still fundamentally falls under the 2SP method category. This approach relies on the "pseudo-range domain" processing strategy, meaning that when the receiver determines its position, it must first obtain pseudo-range information, which prevents it from achieving Maximum Likelihood Estimation (MLE). The optimal solution under the DPE framework involves directly accumulating signal energy in the navigation domain. This is done by combining signals from all visible satellites and using the joint accumulation output to obtain the navigation solution through one-step estimation, as illustrated in Figure 3.

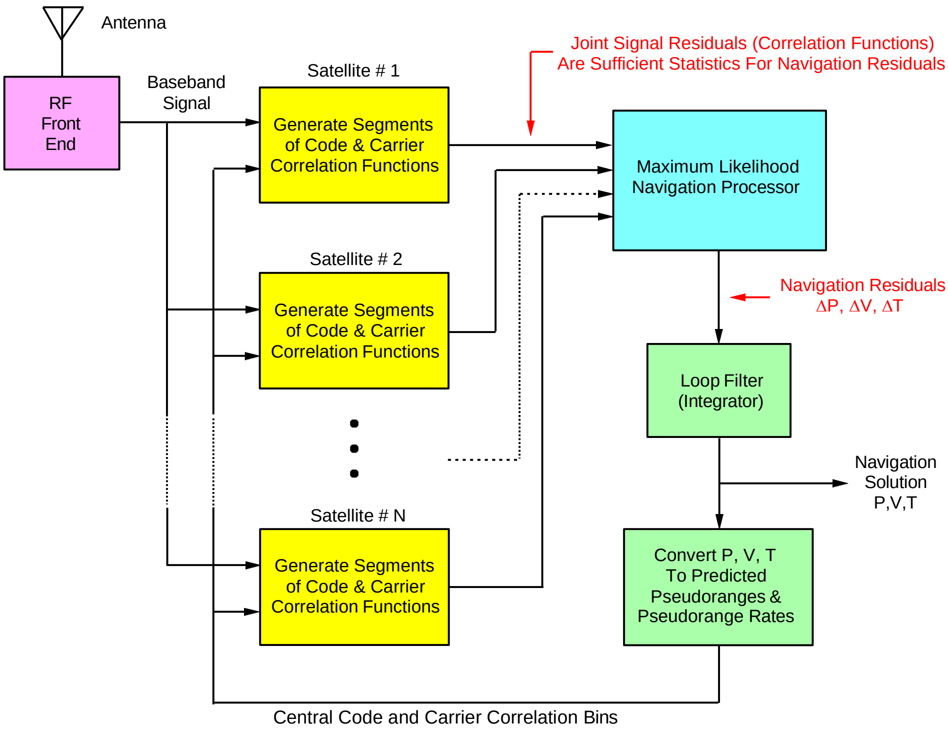

Figure 3. Maximum Likelihood Vector Correlation Function Technique. (*Source:* Reproduced with ION permission)

All visible satellite signals can be used without the need for capturing and tracking, enabling reliable positioning performance even in weak signal environments (Closas P., et.al., 2007). Additionally, the system demonstrates higher sensitivity. The DPE navigation solution is determined through the combined results from multiple channels. As a result, even if some channels are impacted by multipath interference, accurate positioning can still be achieved (Closas P., et.al, 2009a).

A comparison of scalar tracking, vector tracking, and DPE is presented in Table 1. In conclusion, DPE is a one-step method that derives the navigation solution through a single estimation in the navigation domain. This processing strategy is based on the "navigation domain" and is classified as a one-step method. (Closas P., et.al, 2009b) demonstrated through maximum likelihood estimation that the DPE method is the most effective approach.

2. Basic Principles of Direct Position Estimation

As an emerging technology in the field of navigation, DPE has the potential to address the limitations of traditional methods, which often struggle to function effectively in complex environments, such as urban areas with tall buildings and indoor spaces. Consequently, an increasing number of researchers are focusing their efforts on DPE studies. The potential of the DPE approach is being explored in the literature, highlighting its conceptual benefits, analytical advantages, demonstrated improvements, and various processing techniques.

Research has identified the conceptual motivations for DPE (Axelrad, P., et.al, 2011; Brown, 2012; Closas P., et.al, 2017), as well as its analytical advantages (Closas P., et.al 2007 & 2009a; Gusi-Amigó, A., et.al., 2018). Additionally, the documented improvements of DPE-based receivers have been presented in various studies (Chu, 2018; Dampf., et.al., 2017; Ng and Gao, 2016b).

This work aims to contribute to the development of the knowledge base surrounding DPE by discussing practical considerations for DPE-based receiver architecture. In the following sections, we will first introduce the basic principles of DPE. The paper presents the MLE of position within the GNSS framework, introducing the concept of DPE. This work is considered groundbreaking as it proposes the DPE concept and derives it from a theoretical standpoint. Below is a brief overview of the main derivation process for DPE.

Table 1. Scalar tracking, vector tracking, and DPE Comparison (*Source:* Reproduced with ION permission)

	Approach	**Remarks**
Scalar Tracking	a. Estimate intermediate range measurements b. Solve for the navigation solution using intermediate measurements	a. Susceptible to intermediate range estimation errors b. Does not account for inter-link correlations
Vector Tracking	a. Estimates intermediate measurement residuals b. Couples signal tracking and position, velocity, and time (PVT) estimation, so that all channels aid each other by sharing information c. Map intermediate measurement residuals to navigation residuals	a. Susceptible to intermediate residual estimation errors b. Joint optimization across satellites
Direct Position Estimation	a. Maximize cross-correlation of expected GNSS signal reception with received GNSS signal at multiple navigation candidates	a. Direct search b. Joint optimization across satellites

Assume that the receiver antenna has captured K snapshots. The model for the received signal can be expressed as Equation 1 (Zhou Z., 2023):

$$\boldsymbol{x} = \boldsymbol{a}\boldsymbol{D}(\boldsymbol{\theta}) + \boldsymbol{n} \tag{1}$$

In the formula:

$\boldsymbol{x} \in \mathbb{C}^{1xK}$ is the observation vector of the receiver.

$a = [a_1, a_2, \cdots, a_m] \in \mathbb{C}^{1xM}$ where M is the amplitude of the received satellite signal.

$\boldsymbol{D}(\boldsymbol{\theta}) = [\boldsymbol{d}(t_0), \boldsymbol{d}(t_1, \cdots, \boldsymbol{d}(t_{k-1})] \in \mathbb{C}^{MxK}$ is the basis function matrix.

$\boldsymbol{d}(t) = [d_1, d_2, \cdots, d_M]^T \in \mathbb{C}^{Mx1}$, where $d_i = s_i(t - \tau_i)\mathrm{e}^{\mathrm{j}2\pi f_{di}t}$.

$\theta = \left[\tau^{\mathrm{T}}, \mathrm{f}_{\mathrm{d}}^{\mathrm{T}}\right]^{\mathrm{T}}$, τ is code phase, $\boldsymbol{f}_d$ is the carrier Doppler frequency shift.

$n \in \mathbb{C}^{1xK}$ is zero mean additivity Gaussian white noise with variance σ_n^2.

Considering the MLE as equivalent to the solution obtained through least squares, and under the assumption of zero-mean additive Gaussian white noise, maximizing the observed likelihood cost function is equivalent to minimizing Equation 2.

$$\Lambda(\boldsymbol{a}, \boldsymbol{\tau}, \boldsymbol{f}_d) \triangleq \Lambda(\boldsymbol{a}, \boldsymbol{\theta}) = \frac{1}{K}\|\boldsymbol{x} - \boldsymbol{a}\boldsymbol{D}(\boldsymbol{\theta})\|^2 \tag{2}$$

Equations 3, 4, 5 and 6 are related to each other through the definition of cross-correlation.

$$\hat{\boldsymbol{r}}_{\mathrm{xx}} = \frac{1}{K}\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}} \tag{3}$$

$$\hat{\boldsymbol{R}}_{\mathrm{xd}}(\boldsymbol{\theta}) = \frac{1}{K}\boldsymbol{x}\boldsymbol{D}^{\mathrm{H}}(\boldsymbol{\theta}) \tag{4}$$

$$\hat{\boldsymbol{R}}_{\mathrm{dx}}(\boldsymbol{\theta}) = \hat{\boldsymbol{R}}_{\mathrm{xd}}^{\mathrm{H}}(\boldsymbol{\theta}) \tag{5}$$

$$\hat{\boldsymbol{R}}_{\mathrm{dd}}(\boldsymbol{\theta}) = \hat{\boldsymbol{R}}_{\mathrm{xd}}^{\mathrm{H}}(\boldsymbol{\theta}) \tag{6}$$

In the formula:

$\hat{\boldsymbol{r}}_{\mathrm{xx}}$ is the autocorrelation result of the receiver observation.

$\hat{\boldsymbol{R}}_{\mathrm{xd}}(\boldsymbol{\theta})$ is the cross-correlation function between the observed value and the true value.

$\hat{\boldsymbol{R}}_{\mathrm{dd}}(\boldsymbol{\theta})$ is the autocorrelation function of the observed values.

So we can get directly MLE Amplitude as Equation 7

$$\hat{\boldsymbol{a}}_{ML} = \hat{\boldsymbol{R}}_{xd}(\boldsymbol{\theta})\hat{\boldsymbol{R}}_{dd}^{-1}(\boldsymbol{\theta})|_{\tau=\hat{\tau}_{ML}, f_d=\hat{f}_{d_{ML}}} \quad (7)$$

An example of the shape of $\hat{\boldsymbol{a}}_{ML}$ is illustrated in Figure 4. Since the maximum likelihood estimator (MLE) is consistent, the cost function is optimized at the true PVT solution. It has been demonstrated that DPE offers additional robustness in challenging scenarios (Closas, P.,et.al, 2007 and 2015).

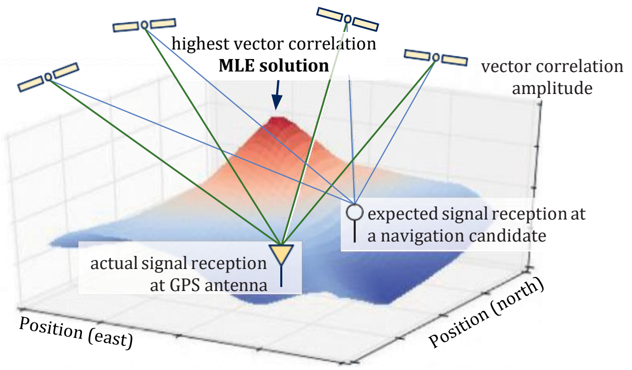

Figure 4. The manifold of the vector correlation amplitudes $\hat{\boldsymbol{a}}_{ML}$, shown in the east-north position domain. *Source:* Reproduced with ION permission

Furthermore, substituting equation (7) into equation (2) gives us equation (8).

$$\begin{cases} \hat{\boldsymbol{\tau}}_{\mathrm{ML}}, \hat{\boldsymbol{f}}_{d_{\mathrm{ML}}} = \underset{v=[\tau^T, f_d^T]^T}{\operatorname{argmin}} \{\Lambda(\boldsymbol{\theta})\} \\ = \underset{\tau, f_d}{\operatorname{argmin}}\{\hat{\mathbf{r}}_{xx} - \hat{\boldsymbol{R}}_{xd}(\boldsymbol{\tau}, \boldsymbol{f}_d)\hat{R}_{dd}^{-1}(\boldsymbol{\tau}, \boldsymbol{f}_d)\hat{R}_{xd}^{H}(\boldsymbol{\tau}, \boldsymbol{f}_d)\} \end{cases} \quad (8)$$

With the known satellite ephemeris, code phase, and carrier Doppler, then we can find the position, speed, clock error, and clock drift of the receiver as Equation 9:

$$\begin{cases} \rho_i = c\tau_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + c(\delta t - \delta t_i) + \varepsilon_i \\ f_{di} = \left(\frac{v_i - v}{c}\boldsymbol{u}_i\right) f_c \\ \boldsymbol{u}_i = \frac{\boldsymbol{p}_i - \boldsymbol{p}}{\|\boldsymbol{p}_i - \boldsymbol{p}\|} \end{cases} \quad (9)$$

where:

ρ_i for the receiver and the i^{th} pseudo-range of satellites

c is the speed of light.

δt and δt_i for the receiver and the i^{th} clock error of satellites.

f_{di} is the Doppler frequency of the i^{th} satellite and receiver pair.

v and v_i represent the receiver and the i^{th} three-dimensional velocity of a satellite.

u_i for the receiver and the i^{th} direction vector of each satellite,

ε is the measurement error.

f_c is the carrier frequency.

$p = (x, y, z)$ and $p_i = (x_i, y_i, z_i)$ is the receiver and the i^{th} three-dimensional position coordinates of each satellite.

Let $\gamma = [p^T, v^T]^T = [x, y, z\ \delta t, \dot{x}, \dot{y}, \dot{z}, \delta\dot{t}]^T$, $(\dot{x}, \dot{y}, \dot{z})$ is the three-dimensional velocity.

If $\delta\dot{t}$ is a clock drift, then there is $\tau \triangleq \tau(\gamma)$, $f_d \triangleq f_d(\gamma)$.

Therefore, the formula (7) can be written as Equation 10:

$$\begin{cases} \hat{\gamma}_{ML} = \underset{\gamma}{\operatorname{argmin}}\{\Lambda(\gamma)\} \\ = \underset{\gamma}{\operatorname{argmin}}\{\hat{r}_{zx} - \hat{R}_{xd}(\gamma)\hat{R}_{dd}^{-1}(\gamma)\hat{R}_{xd}^{H}(\gamma)\} \end{cases} \quad (10)$$

At this point, the traditional positioning based on synchronization parameters has been transformed into a receiver DPE position.

3. Results and Discussions

DPE is a high-sensitivity receiver design, particularly effective in GNSS applications. Unlike traditional methods, the DPE algorithm solves for PVT directly from raw satellite signals, bypassing the need for intermediate quantities like pseudorange and carrier phase

3.1. Performance in Challenging Environments

To evaluate the sensitivity of DPE's performance in challenging environments, it has been demonstrated that DPE performs better in terms of precision and robustness, particularly in multipath channels. This assessment utilized the Land Mobile Satellite Channel Model (LMSCM) outlined in the ITU-R P.681 recommendation (ITU-R P2145-2, 2017), developed by the German Aerospace Center (DLR) (Alexander et al., 2019), as illustrated in Figure 5.

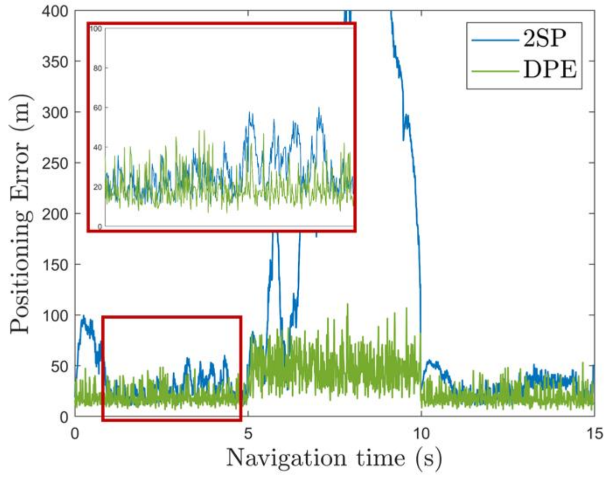

Figure 5. The positioning error of 2SP and DPE under the scenarios simulated by LMSCM during a 15-second navigation recording shows persistent signal contamination from multipath effects, with only three satellites remaining in the on-line-of-sight (LOS). *Source:* Reproduced with ION permission

Another study that has discussed the challenges of estimating GPS signal delays in environments with Multipath propagation has been done (Soubielle, et.al, 2002). In these environments, signals reflect off surfaces before reaching the receiver. Traditional methods, such as the early-late estimator, work well in single-path scenarios but can introduce significant errors (up to 100 meters) in multipath conditions due to biases in delay estimation. The study concludes that the proposed MLE estimator provides a robust solution for GPS signal delay estimation in multipath environments, significantly improving accuracy while keeping implementation costs manageable. The findings highlight the importance of adapting existing algorithms to better handle the complexities introduced by multipath propagation.

3.2. Interference Tolerance

Various Robust Estimators (REs) have been applied in geodetic networks (Koch IE, et.al, 2019). These include M-estimators, which are particularly effective for outlier detection and robust adjustment. M-estimators are a broad class of estimators in statistics, used primarily for robust regression. They generalize MLE by minimizing a sum of functions of the data. The key idea is to reduce the influence of outliers on the estimation process. M-estimators are used in regression analysis, particularly in situations where data may have outliers, or the error distribution is not normal. It also useful in machine learning for developing robust models that do not overfit to the noise in the data.

Computer simulations were conducted to compare the performance of the DPE approach with that of the conventional 2SP positioning method (Closas P., et al., 2017). The objective of these simulations was to validate the theoretical assertions related to the Mean Squared Error (MSE) performance of both positioning estimators under realistic signal conditions. Specifically, the simulations encompassed scenarios where the strength of satellite signals varied, including instances of weak satellite signals and independent degradation effects.

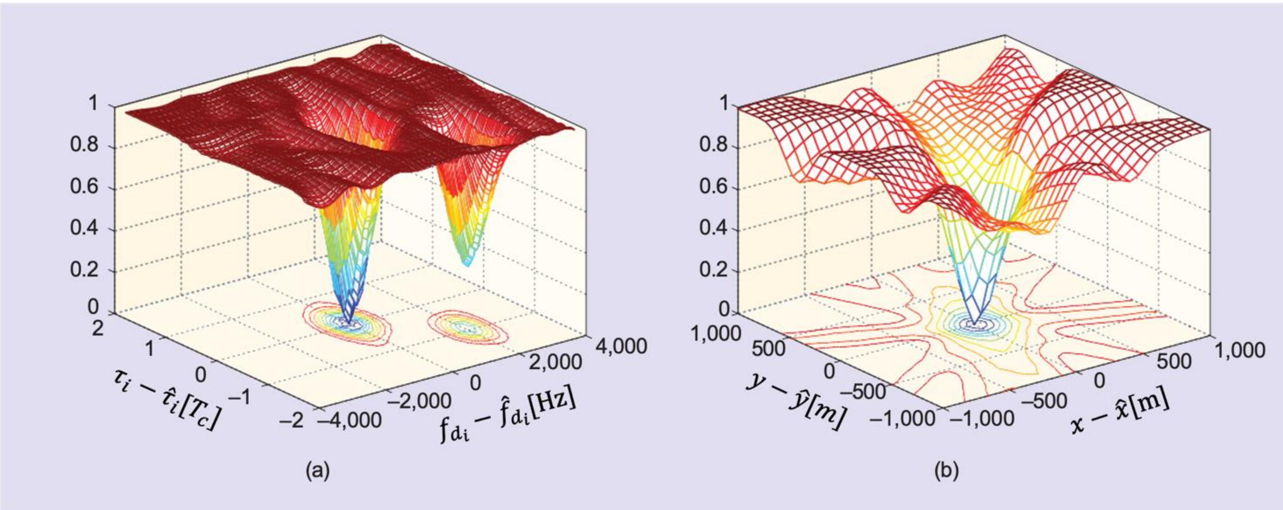

Figure 6. Comparing the cost functions for (a) 2SP and (b) DPE optimization problems shows that, in the presence of a replica for one of the satellites, a strong secondary optimum appears in the 2SP case, while the DPE cost function remains virtually unchanged.. *Source:* Reproduced with ION permission

In Figure 6a and 6b, when a multipath replica is present for one of the satellites the behavior of the cost function differs. Particularly, by adding an echo for one satellite (3 dB lower than the line-of-sight signal/LOSS). Whereas DPE's cost function remains virtually unaltered due to this effect, the two-steps cost function exhibits a strong secondary optimum due to the presence of a correlated

The results indicated that DPE consistently outperformed the conventional method, particularly in low Signal-to-Noise Ratio (SNR) conditions. In these scenarios, the performance of the conventional estimator was significantly degraded due to Multiple Access Interference (MAI) and low-powered signals. Additionally, the simulations demonstrated by (Closas 2009) that the Cramer-Rao Bound (CRB) for DPE was lower than that for the conventional approach, further emphasizing the advantages of DPE in alleviating the challenges posed by weak signals.

On the other hand, the CRB, as a commonly used indicator for evaluating the performance of an estimator, is only accurate in high SNR regions, and will produce high estimation errors in low SNR regions. Therefore, using lower bound of the CRB is not conducive to the discussion and analysis DPE. The performance limit of the method and the traditional 2SP method in weak signal scenarios. Therefore (Gusi-Amigo, 2018) derived additive white Gaussian noise channel and called as Ziv-Zakai Bound (ZZB), and pointed out that for fields with different SNR, traditional weighting matrix obtained from lower bound of the CRB is not optimal for the entire SNR range, while ZZB obtained weighted matrix has better performance. In summary, the ZZB is an effective tool for evaluating the performance of estimators in Additive White Gaussian Noise (AWGN) channels, particularly when traditional bounds such as the Cramér-Rao Bound (CRB) are inadequate. It aids in comprehending the essential limits of estimation accuracy in noisy environments.

3.3. DPE Potential Application in Geodetic Networks

The accurate determination of positions within a geodetic network is crucial for various applications, including land surveying, navigation, and geophysical studies. Traditional methods of position estimation often rely on indirect measurements and complex algorithms, which can introduce errors and increase computational demands

DPE is an effective navigation and positioning technique that employs statistical methods to estimate model parameters. When used in geodetic networks, DPE helps mitigate the impacts of multipath interference and jamming. This method enables the identification and management of outliers, leading to more accurate and reliable positioning. By directly inferring positions from sampled data in geodetic networks, Maximum Likelihood Estimation (MLE) improves the robustness and accuracy of position estimates (Closas, P., 2007).

Recently, a novel grid-based maximum likelihood estimation (MLE) algorithm based on differential pseudorange estimation (DPE) has been developed to significantly reduce computational load. This algorithm utilizes pseudorange measurements to generate a correlogram within a predefined search space (Vicenzo S., et al., 2024).. This method holds promise for applications in surveying, mapping, and geoinformation systems, where precise positioning is crucial. By integrating DPE, which can enhance the accuracy and efficiency of geodetic networks into geodetic practices, the reliability and efficiency of spatial data collection can be significantly improved, paving the way for advancements in geodesy and related fields.

Future work involves creating more efficient methodologies for implementing DPE with reduced computational costs and developing principled methods to integrate diverse information from multiple sources.

4. Conclusion

A robust estimator for a geodetic network is a statistical method that provides reliable parameter estimates even when there are outliers or significant errors in the data. Unlike traditional least-squares methods, which can be heavily influenced by outliers, robust estimators are less sensitive to these anomalies. Techniques such as MLE play a crucial role in maintaining the accuracy and reliability of adjustments in geodetic networks when faced with outliers. These robust methods serve as effective alternatives to traditional least-squares approaches, ultimately enhancing the quality of geodetic data analysis.

5. Competing Interests

The authors declare no conflict of interest in this article.

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