TOTAL FACTOR PRODUCTIVITY AND FRONTIER PRODUCTION FUNCTION

(Produktivitas Faktor Produksi Total dan Fungsi Produksi Frontier)

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Abstract

The total factor productivity became an interesting concept in the measurement of productivity growth. Productivity is a ratio of output to input. The most common measurement of productivity is single factor productivity or partial productivity such as of land, labor, or capital.

A total (factor) productivity is a productivity of all factors of production where the factors are aggregated. In cross-sectional studies this total productivity is a ratio of actual to potential output where the potential output is estimated from ther frontier production function. One of the methods to estimate this frontier function is by using linear programming technique.

The total productivity does not always coincide with a single factor productivity of land (yield), that in the study area the larger farms tend to have higher total productivity than yield.

Ringkasan

Produktivitas total dari semua faktor produksi atau disingkat produktivitas total mulai digunakan oleh para ahli ekonomi pembangunan untuk mengukur pertumbuhan produktivitas. Pengukuran produktivitas yang banyak digunakan adalah merupakan pengukuran produktivitas salah satu faktor produksi, seperti tanah, tenaga manusia atau modal. Hasil penelitian menunjukkan bahwa pertumbuhan produktivitas, dari studi runtun waktu (time (series) pertumbuhan ekonomi, hanya sebagian saja yang dapat diterangkan oleh adanya pertumbuhan produktivitas masing-masig factor produksi, sementara itu bentuk persamaan fungsi produksi tetap untuk jangka waktu tertentu yang agak panjang.

Produktivitas total adalah produktivitas dari semua faktor produksi dimana pengaruh masing-masing faktor produksi terhadap produksi dijumlahkan. Dalam studi dengan data silang waktu (cross-section) pro-

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duktivitas total dapat dinyatakan dalam perbandingan antara produksi aktual (sebenarnya dari masing-masing usahatani) dengan produksi potensial yang dapat dicapai dengan teknologi yang ada. Salah satu cara untuk memperkirakan produksi potensial adalah dengan mencari fungsi produksi frontier dengan cara linear programming.

Produktivitas total tidak selalu sejalan dengan produktivitas parsial salah satu faktor produksi, misalnya dengan produktivitas sumberdaya tanah yang sering dan lazim digunakan dalam mengukur produktivitas pertanian di negara-negara yang kurang berkembang dengan tekanan kepadatan penduduk, selama ada variasi dalam tingkat penggunaan input lain per kesatuan luas tanah. Dari studi ini ditemukan bahwa produktivitas total lebih cenderung meningkat pada luas usahatani yang lebih besar daripada produktivitas tanah.

Total Factor Productivity

The concept of productivity has become an interesting discussion mainly in the measurement of productivity growth, and it was discussed among others by Evsey D. Domar (1961, 1962), Zvi Griliches (1963), etc. The growth models are not only dealing with the measured inputs such as labor, capital, and land, but also technical progress that may relate to input quality. So the production must be the consequence of the quantity and quality of inputs.

Empirical studies showed that the growth of output has been only a part of it that was contributed by the measured inputs and the technological progress. There are still many other factors contributed in this output growth such as education and skill, better management, change in production mix, economies of scale, external economies and many others (Domar 1961).

Productivity is a ratio of output to input. The productivity of labor, capital, or land is the most common measurement on single factor productivity that is the reciprocal of the input coefficient (input per unit of output). All these are partial productivities, in the sense that output is compared with only one input at a time without and explicit recogniation of the changes of other inputs. Thus a given rise in labor input for example, may be caused by substitution of capital for labor, or by the works of other forces, such as technical change, economies of scale, better management, education and so on.

Suppose in the fully integrated industry in Domar's aggregated production model (Domar 1962) is as,

$$Y = C\,(w^{\circ}L + i^{\circ}K)$$

where Y is output, L is labor, K is capital (includes land), w° and i° are real wage and real return to capital, respectively, in the base period, and C is a productivity index. If all variables are expressed ad index numbers with a common base period,

$$\frac{Y}{Y^\circ} = C\left(\alpha \frac{L}{L^\circ} + \beta \frac{K}{K^\circ}\right)$$

where α and β are factor shares of labor and capital, respectively, in the value of output in the base year. The productivity index of C was called a "Total Factor Productivity" by Kendrick (cited in Domar 1962) and it equals unity in the base year.

$$C = \frac{Y}{w^\circ L + i^\circ K} \qquad \text{or} \qquad C = \frac{Y/Y^\circ}{\alpha\,(L/L^\circ) + \beta\,(K/K^\circ)}$$

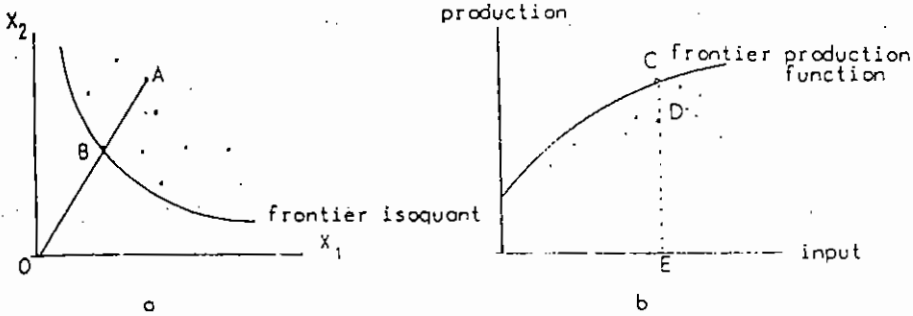
This index absorbs everything left over by convensional defined input and it was called "Residual" by Domar (1961, 1962) in a different meaning from in the statistics. Other term can be used for this index, such as efficiency index, efficiency in the use of resources, Kendrick's index, technical efficiency, production efficiency etc. Unfortunately not all authors agree on the definition of this term (Miller 1984).

However, this efficiency index or total factor productivity has been used by some development economist as a measurement of productivity growth in time series studies. Studies in United States (Griliches 1963) found out that changes in output were atributable to changes in the quantities and qualities of inputs, and to economies of scale. The production function it self remain constant at least over substantial streches of time. Based on this production function the efficiency index or the total factor productivity can be used as a measure of productivity growth, so that the inclusion of more non input factor of production will lead to a substantial reduction in the measured productivity growth as total factor productivity.

In the cross-sectional study of this efficiency in usual sense is supposed to measure a ratio of actual to potential output. This efficiency index of a firm can be evaluated, among others, from an estimated frontier production function that was operationalized through the Farrell's model (Farrell 1957). The potential production frontier is estimated by using the linear programming technique, and taking the highest index of a firm at the frontier function an efficiency of one, the efficiency index equals the ratio of actual to potential output from the same input combinations.

Frontier Production Function

A frontier production function is a production function that is technically most efficient in the sense that their points are in the production possibility set, and there is no way to obtain more output than despicted by this point without using more input. Some farms will be better able to produce than other farms because they have better skill and better endowment, they have better production possibility set.



The frontier production function is maximum production possibility set or the maximum feasibility productivity under farm condition. The frontier production function is operationalized through Farell model (Farell 1957). In case of two kinds of inputs of X_1 and X_2 (fig. a) the frontier isoquant will be the highest production (closest to the origin of 0) in each input combination, or the highest production on frontier production function of figure b. Some farms may be on the technically efficient frontier production function as point B or C, while others lie varying distance away from it (Farell 1957, Timmer 1971). The technical efficiency rating (TER) is OB/OA of fig. a or ED/EC of fig. b. This means that TER is the ratio between actual productivity and the potentially highest productivity. Or in other words the technical efficiency is the output per unit of input where inputs are aggregated in some manner. Or it is also often called the total factor productivity.

This frontier production function as the maximum production possibility set is always have higher or at least equals to the observed actual production for the same level of input use. One of the methods in estimating this frontier is by using linear programming method as :

$$Y_i = AX_{i_1}^{b_1} X_{i_2}^{b_2} \ldots\ldots\ldots X_{i_m}^{b_m} E_i$$

or in logarithm,

$$y_i = b_o + \sum_i b_i x_{ij} + e_i$$

If e_i are constrained to one side of estimated production surface the resulting function is an envelope or frontier function. The equation is estimated as :

$$\hat{b}_o + \sum_j \hat{b}_j x_{ij} = \hat{y}_i \geqslant y_i$$

By setting all $e_i \geqslant 0$, the equation can be written as an equality

$$\hat{b}_o + \sum_j \hat{b}_j x_{ij} - \hat{e}_i = y_i$$

The linear programming technique of estimation is to minimize $\sum_i e_i$ subject to :

$$\hat{b}_o + \sum_j \hat{b}_j x_{ij} \geqslant y_i \qquad (i = 1,2, \ldots\ldots, n)$$

$$\hat{e}_i = \hat{b}_o + \sum_j \hat{b}_j x_{ij} - y_i$$

$$\sum_i e_i = n\hat{b} + \sum_i \sum_j \hat{b}_j x_{ij} - \sum_i y_i \qquad \text{devided by n}$$

$$\frac{\sum_i \ddot{e}_i}{n} = \hat{b}_o + \sum_j \hat{b}_j \bar{x}_j - \bar{y}$$

where $\bar{x}_j$ and $\bar{y}$ are the means of j-th input and output (in logarithm), respectively. Since n and $\bar{y}$ are constant the linear programming structure is :

minimize $\quad \hat{b}_o + \sum_j \hat{b}_j \bar{x}_j$

subject to the constraints of

$$\hat{b}_o + \sum_j \hat{b}_j x_{1j} \geq y_1$$

$$\vdots$$

$$\hat{b}_o + \sum \hat{b}_j x_{nj} \geq y_n$$

Or in matrix form, the objective function is :

$$\begin{bmatrix} 1 & \bar{x}_1 & \dots & \bar{x}_m \end{bmatrix} \begin{bmatrix} b_o \\ b_1 \\ . \\ . \\ b_m \end{bmatrix}$$

and the constraints are

$$\begin{bmatrix} 1 & x_{11} & \dots & x_{1m} \\ . & & & . \\ . & & & . \\ . & & & . \\ 1 & x_{n1} & \dots & x_{nm} \end{bmatrix} \begin{bmatrix} b_o \\ b_1 \\ . \\ . \\ b_m \end{bmatrix} \geqq \begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix}$$

plus the usual non negative constraint of $b_o, b_1, \dots, b_m \geqq 0$.

Or minimize $\bar{x}'b$ subject to $xb \geqq y$ constraints.

Technical efficiency of each sample is measured with the frontier production function. The ratio of the actual production of farm i (Y_i) to the estimated production of farm i ($\hat{y}_i$) from the frontier function estimate gives the technical efficiency rating (TER) of farm i,

$$TER_i = Y_i/\hat{Y}_i$$

In order to avoid the problem of spurious errors in the extreme observation, Timmer (1971) suggest fitting a probabilistic frontier, in which equation

$$\hat{b}_o + \sum_j \hat{b}_j x_{ij} = \hat{y}_i \geqslant y_i$$

must be translated into a probability statement as

$$Pr \left\{ (\hat{b}_o + \sum_j \hat{b}_j x_{ij}) \geqq \hat{y}_i \right\} > p$$

where p is an externally specified probability (e.g. 98%) for which the inequality is to hold. The value of p will be obtained by deleting a percentage of observation on the assumption that they were affected by statistical error, e.g. by deleting 2% of observation which are most efficient (Timmer 1971).

Yield, Total Productivity, and Farm Size

Yield or production per acre or per hectare, is a single factor productivity of land resource that is frequently used to measure the productivity in densely populated of less developed countries (LDCs) due to the most scarce land resource in agricultural production. Recent studies in South America (Brazil, Columbia) and Asia (Philippines, Pakistan, India) found out that output per land area declines as farm size rises (Berry & Cline 1979, Reynold 1975). The small farm sector make better use of its available land than does the large farm sector, largely through applying higher level of other more abundance inputs especially labor input due to the labor market dualism or the dichotomy between the use of family labor on small own farm and hired labor in other large farm. The table show that there are also some evidence that in certain village (Rancaudik) there was inverse relationship between yield or land factor productivity and farm size, although not for Rancaekek.

Hoever, the TER, that is also a measure of productivity of all factors of production that is called total (factor) productivity does not always coincide with the single factor productivity of land resource (yield), and it seems that in general the larger farm had higher total productivity. Therefore this result does not mean to be rejecting the yield-size inverse relationship paradigm as long as land is the most limiting agricultural resources in densely populated of LDCs.

Table. Average Rice Yield And Technical Efficiency Rating (TER) by Farm Size

	Farm Size (hectares)					
	.0-.19	.2-.49	.5-.99	1.0-1.49	1.5-1.99	2.0-
Rice Yield (quintals per hectare)						
Rancaudik 82 — 83 ws		62.62	54.46	52.7	44.29	54.29
Rancaudik 1982 ds		47.79	45.7	37.56	35.12	38.57
Rancaekek 82 — 83 ws	39.75	39.12	37.63	41.72	44.46	
Gadingsari 79 — 80 ws*	19.34	18.67	17.04			
Gadingsari 1979 ds*	21.7	18.94	17.78			
Technical Efficiency Rating (TER)						
Rancaudik 82 — 83 ws		.7673	.6799	.742	.6358	.913
Rancaudik 1982 ds		.5733	.6294	.5884	.4504	.8849
Rancaekek 82 — 83 ws	.8541	.7426	.728	.7099	.8094	
Gadingsari 79 — 80 ws	.8560	.8674	.8593			
Gadingsari 1979 ds	.8786	.8382	.8383			

Note : * Rice yield at Gadingsari Village was conversed in milled rice equivalence.

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