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A Neat Way to Calculate the Gas Velocity from the Ergun Equation in a Packed Bed

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The Ergun equation is often used to describe the pressure drop in a packed bed. This paper presents a method to calculate neatly the gas velocity from the Ergun equation. This method is illustrated using rapid pressure swing adsorption, where flow resistance in the bed is crucial for the successful operation of the process.

Keywords: Darcy's law, Ergun equation, modeling, pressure drop, *and* rapid pressure swing adsorption (RPSA).

INTRODUCTION

The flow resistance in a packed bed is often modeled using the Ergun equation. One example where flow resistance in the bed is crucial for successful operation of the process is *rapid pressure swing adsorption* (RPSA), which is a single packed-bed process used for air separation. It operates with very short cycle times (in the order of seconds) and uses small adsorbent particles (typically 200–700 mm in diameter).

The basic RPSA cycle consists of two steps: (a) *pressurization* and (b) *depressurization.* During *pressurization,* air is fed into the column through a three-way valve. Pressure increases rapidly at the feed end of the column. As feed air flows down the column, nitrogen is preferentially adsorbed on the zeolite 5A adsorbent, resulting in an oxygen-enriched gas phase. In *depressurization*, the feed valve is closed and the exhaust valve at the feed end is opened to atmospheric pressure, resulting in a rapid pressure drop at the feed end of the column, followed by desorption of the adsorbed nitrogen. The gas leaving the exhaust port is enriched with nitrogen. Because there is maximum pressure in the bed during depressurization, a suitable pressure gradient is maintained between this maximum value and the product end of the bed to ensure a continuous product stream throughout the cycle. The momentum balance must also be included in the modeling of RPSA. The local superficial velocity in RPSA varies both with time and position, and can be calculated from the Ergun equation.

The object of this short paper is to present a method to calculate neatly the local superficial gas velocity from the Ergun equation.

GAS FLOW IN PACKED BEDS

In RPSA, the cyclic nature of the process results in an unsteady state gas flow through the packed bed. The momentum balance for the unsteady state gas flow is given by

$$\rho_g\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial z}-J_v u-J_k u^2 \quad (1)$$

where ρ_g is the gas density (kg m^{-3}), t is the time (s), z is an axial coordinate (m), and u is the superficial gas velocity (m s^{-1}). The magnitude of macroscopic inertial forces, the left hand side (LHS) of Eq. (1), is found to be small relative to the right-hand side (RHS) of Eq. (1) and can be neglected (Rousar et al. 1992, Kikkinides and Yang 1993). Eq. (1) then reduces to the steady-state momentum balance for gas flow in packed beds, the Ergun equation

$$\frac{dP}{dz}=-J_v u-J_k u^2 \quad (2)$$

The first term and the second term on the RHS of Eq. (2) represent the *viscous drag losses* and *form drag losses,* respectively. J_v is commonly known as the *bed permeability* (N s m^{-4}). For low values of particle Reynolds number ($Re_p = ud\rho_g/m$), Re_p<5, where the flow is dominated by viscous effects, the contribution of the second term becomes negligible and Eq. (2) reduces to Darcy's law

$$\frac{dP}{dz}=-J_v u \quad (3)$$

For spherical particles and $Re_p < 1000$, J_v and J_k (N s^2 m^{-5}) are given by Ergun (1952) as:

$$J_v=150\frac{\mu(1-\varepsilon_b)^2}{d_p^2\varepsilon_b^3}$$

and

$$J_k=1.75\frac{(1-\varepsilon_b)\rho_g}{d_p^2\varepsilon_b^3} \quad (4)$$

where μ is the gas viscosity (N s m^{-2}), d_p is the particle diameter (m), and ε_b is the bed porosity. MacDonald et al. (1979) suggested that an alternative coefficient value of 180 in J_v gives improved prediction of the pressure-and-flow relationship, and to which this present work subscribes.

LOCAL SUPERFICIAL GAS VELOCITY

The *local superficial velocity* in RPSA is a function both of time and position. Consider first the pressurization step where feed gas flows into the packed bed from the feed end. To calculate the local superficial velocity, Eq. (2) can be rearranged to give the quadratic equation:

$$J_k u^2 + J_v u + (dP/dz) = 0 \quad (5)$$

The two roots of the quadratic equation (5) are often given as:

$$u_1=\frac{-J_v+\sqrt{J_v^2-4J_k(dp/dz)}}{2J_k} \quad (6a)$$

and

$$u_2=\frac{-J_v-\sqrt{J_v^2-4J_k(dp/dz)}}{2J_k} \quad (6b)$$

We reject Eq. (6b) based on the reasoning that when $dP/dz = 0$; that is, when no gas flows into the packed bed, $u_1 = 0$, but $u_2 \neq 0$, which is physically incorrect.

Eq. (6a), however, breaks down when $J_k = 0$; that is, when the Ergun equation reduces to Darcy's law. Further, if J_v^2 is much larger than $4J_k(dp/dz)$, the numerator in the calculations for u_1 will involve the subtraction of nearly equal numbers and may, therefore, introduce a large numerical error. To obtain a more accurate result, the researchers rationalized the numerator of Eq. (6a) to give (Burden and Faires 2001):

$$u_1=\frac{-2(dp/dz)}{J_v+\sqrt{J_v^2-4J_k(dp/dz)}} \quad (7)$$

Thus, Eq. (7), which now involves the addition of nearly equal numbers in the denominator, no longer presents any problems numerically. The researchers then set $u = u_1$. To account for the depressurization step, where gas flows out of the packed bed from the feed end, Eq. (7) is modified to give

$$u = -\frac{2(dp/dz)}{J_v + \sqrt{J_v^2 + 4J_k|dp/dz|}}. \quad (8)$$

Note that when $J_k = 0$; that is, when the Ergun equation reduces to Darcy's law, Eq. (8) yields

$$u = -\frac{1}{J_v}\frac{dp}{dz}. \quad (9)$$

Eq. (7), therefore, is superior than Eq. (6a) as a way of calculating the local superficial gas velocity from the Ergun equation.

CONCLUSION

After the gas flow equation used to model the pressure drop in a packed bed was discussed, one arrives at a neat way to calculate the local superficial gas velocity from the Ergun equation.

This method is better because it avoids (a) any potential numerical error in calculating the velocity from the Ergun equation; and (b) the discontinuity in numerical calculation when the Ergun equation reduces to Darcy's law.

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NOTATION

d_p	particle diameter	m
J_v	bed permeability	N s m^{-4}
J_k	coefficient in the Ergun equation	N s^2 m^{-5}
P	total bed pressure	Pa
t	time	s
u	superficial gas velocity	m s^{-1}
z	axial coordinate	m
ε_b	bed porosity	
ε_p	particle porosity	
ρ_b	bed bulk density	kg m^{-3}
ρ_g	gas density	kg m^{-3}

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